Fuzzy Systems : Soft Computing Course Lecture 35 – 36, notes, slides

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Fuzzy Systems

Soft Computing

Fuzzy systems, topics : Introduction, fuzzy logic, fuzzy system elements - input vector, fuzzification, fuzzy rule base, membership function, fuzzy inferencing, defuzzyfication, and output vector. Classical Logic - statement, symbols, tautology, membership functions from facts, modus ponens and modus tollens; Fuzzy logic - proposition, connectives, quantifiers. Fuzzification, Fuzzy inference - approximate reasoning, generalized modus ponens (GMP), generalized modus tollens (GMT). Fuzzy rule based system – example; Defuzzification - centroid method.

Fuzzy Systems

Soft Computing

Topics

(Lectures 35, 36 2 hours)

Fuzzy Systems

What are Fuzzy Systems ?

- Fuzzy Systems include Fuzzy Logic and Fuzzy Set Theory.
- Knowledge exists in two distinct forms :
  - the Objective knowledge that exists in mathematical form is used in engineering problems; and
  - the Subjective knowledge that exists in linguistic form, usually impossible to quantify.

  Fuzzy Logic can coordinate these two forms of knowledge in a logical way.
- Fuzzy Systems can handle simultaneously the numerical data and linguistic knowledge.
- Fuzzy Systems provide opportunities for modeling of conditions which are inherently imprecisely defined.
- Many real world problems have been modeled, simulated, and replicated with the help of fuzzy systems.
- The applications of Fuzzy Systems are many like : Information retrieval systems, Navigation system, and Robot vision.
- Expert Systems design have become easy because their domains are inherently fuzzy and can now be handled better;

  examples : Decision-support systems, Financial planners, Diagnostic system, and Meteorological system.

1. Introduction

Any system that uses Fuzzy mathematics may be viewed as Fuzzy system.

The Fuzzy Set Theory - membership function, operations, properties and the relations have been described in previous lectures. These are the prerequisites for understanding Fuzzy Systems. The applications of Fuzzy set theory is Fuzzy logic which is covered in this section.

Here the emphasis is on the design of fuzzy system and fuzzy controller in a closed–loop. The specific topics of interest are :

- Fuzzification of input information,
- Fuzzy Inferencing using Fuzzy sets ,
- De-Fuzzification of results from the Reasoning process, and
- Fuzzy controller in a closed–loop.

Fuzzy Inferencing, is the core constituent of a fuzzy system. A block schematic of Fuzzy System is shown in the next slide. Fuzzy Inferencing combines the facts obtained from the Fuzzification with the fuzzy rule base and conducts the Fuzzy Reasoning Process.

- **Fuzzy System**

A block schematic of Fuzzy System is shown below.

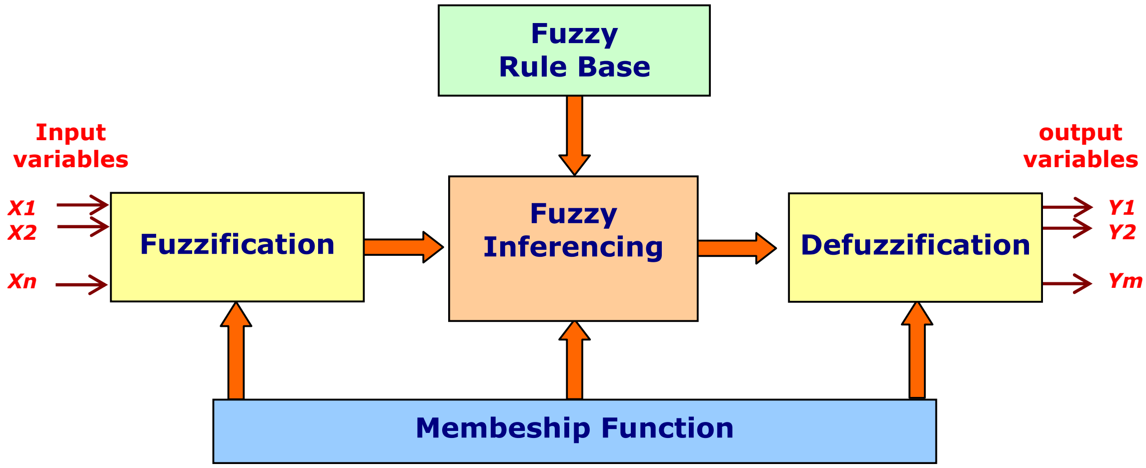


Fig. Elements of Fuzzy System

Fuzzy System elements

- **Input Vector** : $X = [x_1, x_2, \ldots x_n]^T$ are crisp values, which are transformed into fuzzy sets in the fuzzification block.
- **Output Vector** : $Y = [y_1, y_2, \ldots y_m]^T$ comes out from the defuzzification block, which transforms an output fuzzy set back to a crisp value.
- **Fuzzification** : a process of transforming crisp values into grades of membership for linguistic terms, "far", "near", "small" of fuzzy sets.
- **Fuzzy Rule base** : a collection of propositions containing linguistic variables; the rules are expressed in the form:

  If (x is A) AND (y is B) . . . . . . THEN (z is C)

  where **x, y** and **z** represent variables (e.g. distance, size) and **A, B** and **Z** are linguistic variables (e.g. `far', `near', `small').
- **Membership function** : provides a measure of the degree of similarity of elements in the universe of discourse **U** to fuzzy set.
- **Fuzzy Inferencing** : combines the facts obtained from the Fuzzification with the rule base and conducts the Fuzzy reasoning process.
- **Defuzzyfication:** Translate results back to the real world values.

2. Fuzzy Logic

A simple form of logic, called a two-valued logic is the study of "truth tables" and logic circuits. Here the possible values are true as **1**, and false as **0**.

This simple two-valued logic is generalized and called **fuzzy logic** which treats "truth" as a continuous quantity ranging from **0** to **1**.

Definition : Fuzzy logic (FL) is derived from fuzzy set theory dealing with reasoning that is approximate rather than precisely deduced from classical two-valued logic.

- FL is the application of Fuzzy set theory.
- FL allows set membership values to range (inclusively) between **0** and **1**.
- FL is capable of handling inherently imprecise concepts.
- FL allows in linguistic form, the set membership values to imprecise concepts like "**slightly**", "**quite**" and "**very**".

2.1 Classical Logic

Logic is used to represent simple facts. Logic defines the ways of putting symbols together to form **sentences** that represent facts. Sentences are either true or false but not both are called **propositions**.

Examples :

Sentence	Truth value	Is it a Proposition ?
"Grass is green"	"true"	Yes
"2 + 5 = 5"	"false"	Yes
"Close the door"	-	No
"Is it hot out side ?"	-	No
"x > 2"	-	No (since x is not defined)
"x = x"	-	No (don't know what is "x" and "=" mean; "3 = 3" or say "air is equal to air" or "Water is equal to water" has no meaning)

- **Propositional Logic (PL)**

A proposition is a statement - which in English is a declarative sentence and Logic defines the ways of putting symbols together to form sentences that represent facts. Every proposition is either true or false. Propositional logic is also called boolean algebra.

Examples: (a) The sky is blue., (b) Snow is cold. , (c) 12 * 12=144

Propositional logic : It is fundamental to all logic.

‡ Propositions are "Sentences"; either true or false but not both.

‡ A sentence is smallest unit in propositional logic

‡ If proposition is true, then truth value is "true"; else "false"

‡ **Example ;** **Sentence** **"Grass is green";**
Truth value **" true";**
Proposition **"yes"**

Statement, Variables and Symbols

Statement : A simple statement is one that does not contain any other statement as a part. A compound statement is one that has two or more simple statements as parts called components.

Operator or connective : Joins simple statements into compounds, and joins compounds into larger compounds.

Symbols for connectives

assertion	P					"p is true"
nagation	¬p	~	!		NOT	"p is false"
conjunction	$p \wedge q$	·	&&	&	AND	"both p and q are true"
disjunction	P v q	||	|		OR	"either p is true, or q is true, or both "
implication	$p \to q$	⊃	⇒		if . . then	"if p is true, then q is true" " p implies q "
equivalence	↔	≡	⇔		if and only if	"p and q are either both true or both false"

Truth Value

The truth value of a statement is its truth or falsity ,

p is either true or false,

~p is either true or false,

p v q is either true or false, and so on.

"**T**" or "**1**" means "**true**". and

"**F**" or "**0**" means "**false**"

Truth table is a convenient way of showing relationship between several propositions. The truth table for negation, conjunction, disjunction, implication and equivalence are shown below.

p	q	¬p	¬q	p ∧ q	p v q	p→q	p ↔ q	q→p
T	T	F	F	T	T	T	T	T
T	F	F	T	F	T	F	F	T
F	T	T	F	F	T	T	F	F
F	F	T	T	F	F	T	T	T

▪ Tautology

A Tautology is proposition formed by combining other propositions **(p, q, r, . . .)** which is true regardless of truth or falsehood of **p, q, r, ...**

The important tautologies are :

$(p \to q) \leftrightarrow \neg [p \wedge (\neg q)]$ and $(p \to q) \leftrightarrow (\neg p) \vee q$

A proof of these tautologies, using the truth tables are given below.

Tautologies $(p \to q) \leftrightarrow \neg [p \wedge (\neg q)]$ and $(p \to q) \leftrightarrow (\neg p) \vee q$

Table 1: Proof of Tautologies

p	q	$p \to q$	$\neg q$	$p \wedge (\neg q)$	$\neg [p \wedge (\neg q)]$	$\neg p$	$(\neg p) \vee q$
T	T	T	F	F	T	F	T
T	F	F	T	T	F	F	F
F	T	T	F	F	T	T	T
F	F	T	T	F	T	T	T

Note :

1. The entries of two columns $p \to q$ and $\neg [p \wedge (\neg q)]$ are identical, proves the tautology. Similarly, the entries of two columns $p \to q$ and $(\neg p) \vee q$ are identical, proves the other tautology.
2. The importance of these tautologies is that they express the membership function for $p \to q$ in terms of membership functions of either propositions **p** and $\neg q$ or $\neg p$ and **q**.

- **Equivalences**

Between Logic , Set theory and Boolean algebra.

Some mathematical equivalence between Logic and Set theory and the correspondence between Logic and Boolean algebra **(0, 1)** are given below.

Logic	Boolean Algebra (0, 1)	Set theory
T	1	
F	0	
∧	x	∩ , ∩
∨	+	∪ , U
¬	′ ie complement	(‾)
↔	=	
p, q, r	a, b, c	

Membership Functions obtain from facts

Consider the facts (the two tautologies)

$$(p \rightarrow q) \leftrightarrow \neg [p \wedge (\neg q)] \quad \text{and} \quad (p \rightarrow q) \leftrightarrow (\neg p) \vee q$$

Using these facts and the equivalence between logic and set theory, we can obtain membership functions for $\mu_{p \rightarrow q}(x, y)$.

From 1st fact : $\mu_{p \rightarrow q}(x, y) = 1 - \mu_{p \cap \bar{q}}(x, y)$

$= 1 - \min [\mu_p(x), 1 - \mu_q(y)]$ **Eq (1)**

From 2nd fact : $\mu_{p \rightarrow q}(x, y) = 1 - \mu_{\bar{p} \cup q}(x, y)$

$= \max [1 - \mu_p(x), \mu_q(y)]$ **Eq (2)**

Boolean truth table below shows the validation membership functions

Table-2 : Validation of Eq (1) and Eq (2)

$\mu_p(x)$	$\mu_q(y)$	$1 - \mu_p(x)$	$1 - \mu_q(y)$	$\max [1 - \mu_p(x), \mu_q(y)]$	$1 - \min [\mu_p(x), 1 - \mu_q(y)]$
1	1	0	0	1	1
1	0	0	1	0	0
0	1	1	0	1	1
0	0	1	1	1	1

Note :

1. Entries in last two columns of this table-2 agrees with the entries in table-1 for $p \rightarrow q$, the proof of tautologies, read **T** as **1** and **F** as **0**.

2. The implication membership functions of Eq.1 and Eq.2 are not the only ones that give agreement with $p \rightarrow q$. The others are :

$\mu_{p \rightarrow q}(x, y) = 1 - \mu_p(x)(1 - \mu_q(y))$ **Eq (3)**

$\mu_{p \rightarrow q}(x, y) = \min [1, 1 - \mu_p(x) + \mu_q(y)]$ **Eq (4)**

■ Modus Ponens and Modus Tollens

In traditional propositional logic there are two important inference rules, Modus Ponens and Modus Tollens.

Modus Ponens

Premise 1 : **" x is A "**

Premise 2 : **" if x is A then y is B "** ; Consequence : **" y is B "**

Modus Ponens is associated with the implication **" A implies B " [A→B]**

In terms of propositions **p** and **q**, the Modus Ponens is expressed as

$$(p \wedge (p \rightarrow q)) \rightarrow q$$

Modus Tollens

Premise 1 : **" y is not B "**

Premise 2 : **" if x is A then y is B "** ; Consequence : **" x is not A "**

In terms of propositions **p** and **q**, the Modus Tollens is expressed as

$$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$$

2.2 Fuzzy Logic

Like the extension of crisp set theory to fuzzy set theory, the extension of crisp logic is made by replacing the bivalent membership functions of the crisp logic with the fuzzy membership functions.

In crisp logic, the truth value acquired by the proposition are 2-valued, namely true as **1** and false as **0**.

In fuzzy logic, the truth values are multi-valued, as absolute true, partially true, absolute false etc represented numerically as real value between **0** to **1**.

Note : The fuzzy variables in fuzzy sets, fuzzy propositions, fuzzy relations etc are represented usually using symbol **~** as $\tilde{P}$ but for the purpose of easy to write it is always represented as **P** .

• Recaps

01 **Membership function** $\mu_A(x)$ describes the membership of the elements **x** of the base set **X** in the fuzzy set **A** .

02 **Fuzzy Intersection operator** ∩ (AND connective) applied to two fuzzy sets **A** and **B** with the membership functions $\mu_A(x)$ and $\mu_B(x)$ based on min/max operations is $\mu_{A \cap B} = \min [\mu_A(x) , \mu_B(x)] , \quad x \in X$ **(Eq. 01)**

03 **Fuzzy Intersection operator** ∩ (AND connective) applied to two fuzzy sets **A** and **B** with the membership functions $\mu_A(x)$ and $\mu_B(x)$ based on algebraic product is $\mu_{A \cap B} = \mu_A(x) \, \mu_B(x) , \quad x \in X$ **(Eq. 02)**

04 **Fuzzy Union operator** ∪ (OR connective) applied to two fuzzy sets **A** and **B** with the membership functions $\mu_A(x)$ and $\mu_B(x)$ based on min/max operations is $\mu_{A \cup B} = \max [\mu_A(x) , \mu_B(x)] , \quad x \in X$ **(Eq. 03)**

05 **Fuzzy Union operator** ∪ (OR connective) applied to two fuzzy sets **A** and **B** with the membership functions $\mu_A(x)$ and $\mu_B(x)$ based on algebraic sum is $\mu_{A \cup B} = \mu_A(x) + \mu_B(x) - \mu_A(x) \, \mu_B(x) , \quad x \in X$ **(Eq. 04)**

06 **Fuzzy Compliment operator** ($^-$) (NOT operation) applied to fuzzy set **A** with the membership function $\mu_A(x)$ is $\mu_{\bar{A}} = 1 - \mu_A(x) , \quad x \in X$ **(Eq. 05)**

07 **Fuzzy relations** combining two fuzzy sets by connective "min operation" is an operation by cartesian product $R : X \times Y \rightarrow [0 , 1]$.

$\mu_{R(x,y)} = \min[\mu_A(x), \mu_B(y)]$ **(Eq. 06)** or

$\mu_{R(x,y)} = \mu_A(x) \, \mu_B(y)$ **(Eq. 07)**

Example : Relation **R** between fruit colour **x** and maturity grade **y** characterized by base set linguistic colorset **X = {green, yellow, red}** maturity grade as **Y = {verdant, half-mature, mature}**

$R \triangleq$

x \ Y	V	h-m	m
G	1	0.5	0.0
Y	0.3	1	0.4
R	0	0.2	1

08 **Max-Min Composition -** combines the fuzzy relations variables, say (x , y) and (y , z) ; $x \in A , \; y \in B , \; z \in C$.

consider the relations :

$R_1(x , y) = \{ ((x , y) , \mu_{R1} (x , y)) \mid (x , y) \in A \times B \}$

$R_2(y , z) = \{ ((y , y) , \mu_{R1} (y , z)) \mid (y , z) \in B \times C \}$

The domain of R_1 is $A \times B$ and the domain of R_2 is $B \times C$

max-min composition denoted by $R_1 \circ R_2$ with membership function $\mu_{R1 \circ R2}$

$$R_1 \circ R_2 = \{ ((x , z) , \max_y (\min (\mu_{R1} (x , y) , \mu_{R2} (y , z))))\} , \quad (x , z) \in A \times C , \; y \in B$$ **(Eq. 08)**

Thus $R_1 \circ R_2$ is relation in the domain $A \times C$

- **Fuzzy Propositional**

A fuzzy proposition is a statement **P** which acquires a fuzzy truth value **T(P)** .

Example :

P : Ram is honest

T(P) = 0.8 , means **P** is partially true.

T(P) = 1 , means **P** is absolutely true.

- **Fuzzy Connectives**

The fuzzy logic is similar to crisp logic supported by connectives.

Table below illustrates the definitions of fuzzy connectives.

Table : Fuzzy Connectves

Connective	Symbols	Usage	Definition
Nagation	¬	¬ P	1 – T(P)
Disjuction	∨	P ∨ Q	Max[T(P) , T(Q)]
Conjuction	∧	P ∧ Q	min[T(P) , T(Q)]
Implication	⇒	P ⇒ Q	¬P ∨ Q = max (1-T(P), T(Q)]

Here **P , Q** are fuzzy proposition and **T(P) , T(Q)** are their truth values.

- the **P** and **Q** are related by the ⇒ operator are known as antecedents and consequent respectively.
- as crisp logic, here in fuzzy logic also the operator ⇒ represents **IF-THEN** statement like,

  IF x is A THEN y is B, is equivalent to

  R = (A x B) U (¬ A x Y)

  the membership function of **R** is given by

  $\mu_R (x , y) = \max [\min (\mu_A (x) , \mu_B (y)) , 1 - \mu_A (x)]$

- For the compound implication statement like

  IF x is A THEN y is B, ELSE y is C is equivalent to

  R = (A x B) U (¬ A x C)

  the membership function of **R** is given by

  $\mu_R (x , y) = \max [\min (\mu_A (x) , \mu_B (y)) , \min (1 - \mu_A (x), \mu_C (y))]$

Example 1 : (Ref : Previous slide)

P : Mary is efficient , T(P) = 0.8 ,

Q : Ram is efficient , T(Q) = 0.65 ,

¬ P : Mary is efficient , T(¬ P) = 1 − T(P) = 1− 0.8 = 0.2

P ∧ Q : Mary is efficient and so is Ram, i.e.

T(P ∧ Q) = min (T(P), T(Q)) = min (0.8, 0.65)) = 0.65

P ∨ Q : Either Mary or Ram is efficient i.e.

T(P ∨ Q) = max (T(P), T(Q)) = max (0.8, 0.65)) = 0.8

P ⇒ Q : If Mary is efficient then so is Ram, i.e.

T(P ⇒ Q) = max (1− T(P), T(Q)) = max (0.2, 0.65)) = 0.65

Example 2 : (Ref : Previous slide on fuzzy connective)

Let **X = {a, b, c, d}** ,

A = {(a, 0) (b, 0.8) (c, 0.6) (d, 1)}

B = {(1, 0.2) (2, 1) (3, 0.8) (4, 0)}

C = {(1, 0) (2, 0.4) (3, 1) (4, 0.8)}

Y = { 1, 2, 3, 4} the universe of discourse could be viewed as

{ (1, 1) (2, 1) (3, 1) (4, 1) }

i.e., a fuzzy set all of whose elements x have $\mu(x) = 1$

Determine the implication relations

(i) If x is A THEN y is B

(ii) If x is A THEN y is B Else y is C

Solution

To determine implication relations (i) compute :

The operator ⇒ represents **IF-THEN** statement like,

IF x is A THEN y is B, is equivalent to **R = (A x B) U (¬ A x Y)** and

the membership function **R** is given by

$$\mu_R (x , y) = \max [\min (\mu_A (x) , \mu_B (y)) , 1 - \mu_A (x)]$$

Fuzzy Intersection A x B is defined as :

for all x in the set X,

(A ∩ B)(x) = min [A(x), B(x)],

Fuzzy Intersection ¬A x Y is defined as :

for all x in the set X

(¬A ∩ Y)(x) = min [A(x), Y(x)],

A x B =

A \ B	1	2	3	4
a	0	0	0	0
b	0.2	0.8	0.8	0
c	0.2	0.6	0.6	0
d	0.2	1	0.8	0

¬A x Y =

A \ y	1	2	3	4
a	1	1	1	1
b	0.2	0.2	0.2	0.2
c	0.4	0.4	0.4	0.4
d	0	0	0	0

Fuzzy Union is defined as **(A ∪ B)(x) = max [A(x), B(x)] for all x ∈ X**

Therefore **R = (A x B) U (¬ A x Y)** gives

R =

x \ y	1	2	3	4
a	1	1	1	1
b	0.2	0.8	0.8	0
c	0.4	0.6	0.6	0.4
d	0.2	1	0.8	0

This represents **If x is A THEN y is B** ie **T(A ⇒ B) = max (1- T(A), T(B))**

To determine implication relations (ii) compute : (Ref : Previous slide)

Given **X = {a, b, c, d}** ,

A = {(a, 0) (b, 0.8) (c, 0.6) (d, 1)}

B = {(1, 0.2) (2, 1) (3, 0.8) (4, 0)}

C = {(1, 0) (2, 0.4) (3, 1) (4, 0.8)}

Here, the operator ⇒ represents **IF-THEN-ELSE** statement like,

IF x is A THEN y is B Else y is C, is equivalent to

R = (A x B) U (¬ A x C) and

the membership function of **R** is given by

$$\mu_R(x, y) = \max[\min(\mu_A(x), \mu_B(y)), \min(1 - \mu_A(x), \mu_C(y))]$$

Fuzzy Intersection A x B is defined as :
for all x in the set X,
(A ∩ B)(x) = min [A(x), B(x)],

A x B =

A \ B	1	2	3	4
a	0	0	0	0
b	0.2	0.8	0.8	0
c	0.2	0.6	0.6	0
d	0.2	1	0.8	0

Fuzzy Intersection ¬A x Y is defined as :
for all x in the set X
(¬A ∩ C)(x) = min [A(x), C(x)],

¬A x C =

A \ y	1	2	3	4
a	0	0.4	1	0.8
b	0.2	0.2	0.2	0.2
c	0.4	0.4	0.4	0.4
d	0	0	0	0

Fuzzy Union is defined as **(A ∪ B)(x) = max [A(x), B(x)] for all x ∈ X**

Therefore **R = (A x B) U (¬ A x C) gives**

R =

x \ y	1	2	3	4
a	1	1	1	1
b	0.2	0.8	0.8	0
c	0.4	0.6	0.6	0.4
d	0.2	1	0.8	0

This represents **If x is A THEN y is B Else y is C**

- **Fuzzy Quantifiers**

In crisp logic, the predicates are quantified by quantifiers.

Similarly, in fuzzy logic the propositions are quantified by quantifiers.

There are two classes of fuzzy quantifiers :

- Absolute quantifiers and
- Relative quantifiers

Examples :

Absolute quantifiers	Relative quantifiers
round about 250	almost
much greater than 6	about
some where around 20	most

3. Fuzzification

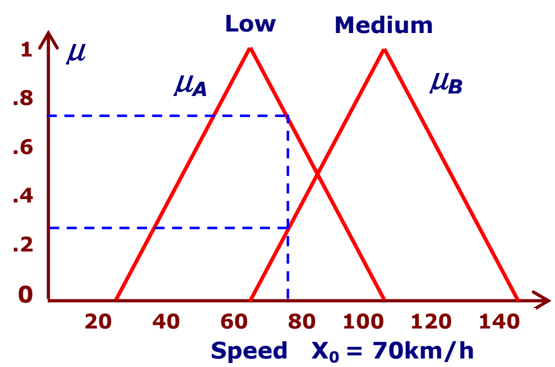
The fuzzification is a process of transforming crisp values into grades of membership for linguistic terms of fuzzy sets.

The purpose is to allow a fuzzy condition in a rule to be interpreted.

- **Fuzzification of the car speed**

Example 1 : Speed X_0 = 70km/h

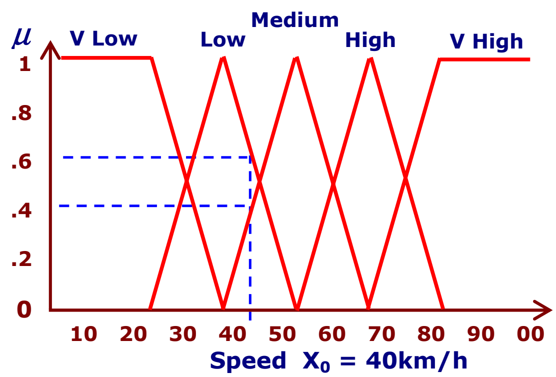
Fig below shows the fuzzification of the car speed to characterize a low and a medium speed fuzzy set.



Characterizing two grades, low and medium speed fuzzy set

Given car speed value **X_0=70km/h** : grade **$\mu_A(x_0)$ = 0.75** belongs to fuzzy low, and grade **$\mu_B(x_0)$ = 0.25** belongs to fuzzy medium

Example 2 : Speed X_0 = 40km/h



Characterizing five grades, Very low, low, medium, high and very high speed fuzzy set

Given car speed value **X_0=40km/h** : grade **$\mu_A(x_0)$ = 0.6** belongs to fuzzy low, and grade **$\mu_B(x_0)$ = 0.4** belongs to fuzzy medium.

4. Fuzzy Inference

Fuzzy Inferencing is the core element of a fuzzy system.

Fuzzy Inferencing combines - the facts obtained from the fuzzification with the rule base, and then conducts the fuzzy reasoning process.

Fuzzy Inference is also known as **approximate reasoning.**

Fuzzy Inference is computational procedures used for evaluating linguistic descriptions. Two important inferring procedures are

- Generalized Modus Ponens (GMP)
- Generalized Modus Tollens (GMT)

- **Generalized Modus Ponens (GMP)**

This is formally stated as

If x is A THEN y is B

x is ¬A

---

y is ¬B

where **A , B , ¬A , ¬B** are fuzzy terms.

Note : Every fuzzy linguistic statements above the line is analytically known and what is below the line is analytically unknown.

To compute the membership function **¬B** , the max-min composition of fuzzy set **¬A** with **R(x , y)** which is the known implication relation (**IF-THEN**) is used. i.e. $\neg B = \neg A \circ R(x, y)$

In terms of membership function

$\mu_{\neg B}(y) = \max(\min(\mu_{\neg A}(x), \mu_R(x, y)))$ where

$\mu_{\neg A}(x)$ is the membership function of **¬A** ,

$\mu_R(x, y)$ is the membership function of the implication relation and

$\mu_{\neg B}(y)$ is the membership function of **¬B**

- **Generalized Modus Tollens (GMT)**

This is formally stated as

If x is A THEN y is B

y is ¬B

---

x is ¬A

where **A , B , ¬A , ¬B** are fuzzy terms.

Note : Every fuzzy linguistic statements above the line is analytically known and what is below the line is analytically unknown.

To compute the membership function **¬A** , the max-min composition of fuzzy set **¬B** with **R(x , y)** which is the known implication relation (**IF-THEN**) is used. i.e. $\neg A = \neg B \circ R(x, y)$

In terms of membership function

$\mu_{\neg A}(y) = \max(\min(\mu_{\neg B}(x), \mu_R(x, y)))$ where

$\mu_{\neg B}(x)$ is the membership function of **¬B** ,

$\mu_R(x, y)$ is the membership function of the implication relation and

$\mu_{\neg A}(y)$ is the membership function of **¬A**

Example :

Apply the fuzzy Modus Ponens rules to deduce Rotation is quite slow?

Given **:**

(i) If the temperature is high then then the rotation is slow.

(ii) The temperature is very high.

Let **H (High) , VH (Very High) , S (Slow) and QS (Quite Slow)** indicate the associated fuzzy sets.

Let the set for temperatures be **X = {30, 40, 50, 60, 70, 80, 90, 100}** , and

Let the set of rotations per minute be **Y = {10, 20, 30, 40, 50, 60}** and

H = {(70, 1) (80, 1) (90, 0.3)}

VH = {(90, 0.9) (100, 1)}

QS = {10, 1) (20, 08) }

S = {(30, 0.8) (40, 1) (50, 0.6)}

To derive **R(x, y)** representing the implication relation (i) above, compute

R (x, y) = max (H x S , ¬ H x Y)

H x S =

	10	20	30	40	50	60
30	0	0	0	0	0	0
40	0	0	0	0	0	0
50	0	0	0	0	0	0
60	0	0	0	0	0	0
70	0	0	0.8	1	0.6	0
80	0	0	0.8	1	0.6	0
90	0	0	0.3	0.3	0.3	0
100	0	0	0	0	0	0

H x Y =

	10	20	30	40	50	60
30	1	1	1	1	1	1
40	1	1	1	1	1	1
50	1	1	1	1	1	1
60	1	1	1	1	1	1
70	0	0	0	0	0	0
80	0	0	0	0	0	0
90	0.7	0.7	0.7	0.7	0.7	0.7
100	1	1	1	1	1	1

[Continued from previous slide]

R(x,Y) =

	10	20	30	40	50	60
30	1	1	1	1	1	1
40	1	1	1	1	1	1
50	1	1	1	1	1	1
60	1	1	1	1	1	1
70	0	0	0.8	1	0.6	0
80	0	0	0.8	1	0.6	0
90	0.7	0.7	0.7	0.7	0.7	0.7
100	1	1	1	1	1	1

To deduce Rotation is quite slow, we make use of the composition rule

QS = VH o R (x, y)

= [0 0 0 0 0 0 0.9 1] x

	10	20	30	40	50	60
30	1	1	1	1	1	1
40	1	1	1	1	1	1
50	1	1	1	1	1	1
60	1	1	1	1	1	1
70	0	0	0	0	0	0
80	0	0	0	0	0	0
90	0.7	0.7	0.7	0.7	0.7	0.7
100	1	1	1	1	1	1

= [1 1 1 1 1 1]

5. Fuzzy Rule Based System

The fuzzy linguistic descriptions are formal representation of systems made through fuzzy IF-THEN rule. They encode knowledge about a system in statements of the form :

IF (a set of conditions) are satisfied THEN (a set of consequents) can be inferred.

IF (x_1 is A_1, x_2 is A_2, xn is An) THEN (y_1 is B_1, y_2 is B_2, yn is Bn)

where linguistic variables x_i, y_j take the values of fuzzy sets A_i and B_j respectively.

Example :

IF there is *"heavy"* rain and *"strong"* winds
THEN there must *"severe"* flood warnings.

Here, **heavy** , **strong** , and **severe** are fuzzy sets qualifying the variables *rain, wind,* and *flood* warnings respectively.

A collection of rules referring to a particular system is known as a fuzzy rule base. If the conclusion **C** to be drawn from a rule base **R** is the conjunction of all the individual consequents C_i of each rule , then

$C = C_1 \cap C_2 \cap \ldots \cap C_n$ where

$\mu_c(y) = \min(\mu_{c1}(y), \mu_{c2}(y), \mu_{cn}(y)), \quad \forall y \in Y$

where **Y** is universe of discourse.

On the other hand, if the conclusion **C** to be drawn from a rule base **R** is the disjunction of the individual consequents of each rule, then

$C = C_1 \cup C_2 \cup \ldots \cup C_n$ where

$\mu_c(y) = \max(\mu_{c1}(y), \mu_{c2}(y), \mu_{cn}(y)), \quad \forall y \in Y$ where

Y is universe of discourse.

6. Defuzzification

In many situations, for a system whose output is fuzzy, it is easier to take a crisp decision if the output is represented as a single quantity. This conversion of a single crisp value is called Defuzzification.

Defuzzification is the reverse process of fuzzification.

The typical Defuzzification methods are

- Centroid method,
- Center of sums,
- Mean of maxima.

Centroid method

It is also known as the "center of gravity" of area method.

It obtains the centre of area **(x*)** occupied by the fuzzy set .

For discrete membership function, it is given by

$$x^* = \frac{\sum_{i=1}^{n} x_i \, \mu(x_i)}{\sum_{i=1}^{n} \mu(x_i)} \quad \text{where}$$

n represents the number elements in the sample, and

x_i are the elements, and

$\mu(x_i)$ is the membership function.

7 References : Textbooks

1. *"Neural Network, Fuzzy Logic, and Genetic Algorithms - Synthesis and Applications", by S. Rajasekaran and G.A. Vijayalaksmi Pai, (2005), Prentice Hall, Chapter 7, page 187-221.*

2. *"Soft Computing and Intelligent Systems Design - Theory, Tools and Applications", by Fakhreddine karray and Clarence de Silva (2004), Addison Wesley, chapter 3, page 137-200.*

3. *"Fuzzy Sets and Fuzzy Logic: Theory and Applications", by George J. Klir and Bo Yuan, (1995), Prentice Hall, Chapter 12-17, page 327-466.*

4. *"Introduction To Fuzzy Sets And Fuzzy Logic", by M Ganesh, (2008), Prentice-hall, Chapter 9-10, page 169- 233.*

5. *"Fuzzy Logic: Intelligence, Control, and Information", by John Yen, Reza Langari, (1999), Prentice Hall, Chapter 8-13, page 183-380.*

6. *"Fuzzy Logic with Engineering Applications", by Timothy Ross, (2004), John Wiley & Sons Inc, Chapter 5-15 , page 120-603.*

7. *"Fuzzy Logic and Neuro Fuzzy Applications Explained", by Constantin Von Altrock, (1995), Prentice Hall, Chapter 3-8, page 29-321.*

8. *Related documents from open source, mainly internet. An exhaustive list is being prepared for inclusion at a later date.*