



POSITION OF EARTH ON CELESTIAL SPHERE AT INPUT UNIVERSAL TIME (UT)

RC Chakraborty (Retd), Former Director, DRDO, Delhi & Visiting Professor, JUET, Guna,
www.myreaders.info, rcchak@gmail.com,

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POSITION OF EARTH ON CELESTIAL SPHERE AT INPUT UNIVERSAL TIME (UT).

Earth is a sphere, the third planet from the Sun and the fifth largest of the eight planets in the Solar System.

Planets order from the Sun : Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune. (Ref <http://nineplanets.org/>)

Earth Rotates on its axis passing through the North and South Poles. The rotation is counterclockwise looking down at North Pole.

The time for Earth to make a complete rotation is approximately 24 hours (exactly 23.9344699 hours or 23 hours, 56 minutes, 4.0916 seconds).

This rotation results daytime in area facing Sun and nighttime in area facing away from Sun. Since we are on Earth, we do not sense its rotation, but experience by observing the relative motion of the Sun (like from a moving vehicle we see the surroundings move).

Earth Revolves around Sun in a counterclockwise direction. The complete orbit (360 deg) is one sidereal year, occurs every 365.256363 mean solar days.

The earth's orbit around the sun is not a circle, it is slightly elliptical. Therefore, distance between earth and sun varies throughout the year.

The Earth Orbit Characteristics, at Epoch J2000, means Y 2000, M 01, D 01, H 12.00, JD = 2451545.00, (Ref http://en.wikipedia.org/wiki/Planet_of_Water)

Eccentricity	0.01671123	Inclination	7.155 deg to Sun's equator	Longitude of ascending node	348.73936 deg
Mean anomaly	357.51716	Orbital period	365.256363004 days	Argument of perihelion	114.20783 deg
Aphelion	152,098,232 km	Perihelion	147,098,290 km	Average orbital speed	107,200 km/h
Semi-major axis	149,598,261 km	Average distance(AS)	149,597,870.700 km from sun		

Note : Ambiguity exist in values reported for Earth to Sun distances, that are mean, average, maximum, minimum, semi-major axis, aphelion, perihelion.

Move on to Compute the Position of Earth on Celestial Sphere at input Universal Time (UT).

At any instant, first need to Compute **Position of Sun** on celestial sphere and then at same instant Compute **Position of Earth** on celestial sphere.

For the Position of Sun on celestial sphere, much has been computed / illustrated in previous section.

The Position of Earth on celestial sphere is characterized by computing around 120 orbital parameters.

The number is large, because some parameters are computed using more than one model equation, that require different inputs.

This helps in validation of results and understanding the different input considerations.

The Orbital Parameters that Characterize the Position of Earth on Celestial Sphere, are put into following groups :

01. GST Greenwich sidereal time and GHA Greenwich hour angle in 0 to 360 deg, at input UT time YY MM DD HH.
02. Earth, Log in 0 to 360 deg and Lat in +ve or -ve in 0 to 90 deg, pointing to Sun Ecliptic Log (Lsun) at time input UT.
03. LST Local sidereal time using GST over three longitudes, Greenwich log, Sun mean log (Lmean), & Sun ecliptic log (Lsun) .
04. ST0 sidereal time over Greenwich longitude = 0.0, at time input Year JAN day 1 hr 00.
05. ST sidereal time, at time input UT, over three log, Greenwich log, Sun mean log (Lmean), and Sun ecliptic log (Lsun).
06. H hour angle in 0 to 360 deg using ST over five longitudes, Greenwich, Lmean, Lsun, Earth Sub Sun point SS, Earth Observation point EP, at time input UT.
07. Delta E is Equation of Time in seconds, using p_julian_day, n_sun, w_sun at time input UT.
08. GST Greenwich sidereal time, and GHA Greenwich hour angle 0 to 360 deg at time when earth is at perihelion.
09. ST sidereal time & MST mean sidereal time at different instances, using Earth mean motion rev per day and julian century days from YY 2000_JAN_1_hr_1200.
10. Earth orbit radius, sub sun point on Earth surface & related parameters, using SMA, e_sun, T_sun, w_sun etc.
11. Earth center(EC) to Sun center(SC) Range Vector[rp, rq, r] in PQW frame (perifocal coordinate system).
12. Transform_1 Earth position EC to SC Range Vector[rp, rq] in PQW frame To Range Vector[rI, rJ, rK] in IJK frame (inertial system cord).
13. Transform_2 Earth point EP(lat, log, hgt) To EC to SC Range Vector[RI, RJ, RK, R] in IJK frame.
14. Transform_3 Earth position EC to SC Range Vectors [rI rJ rK] & [RI RJ RK] To EP to SC Range Vector[rvI, rvJ, rvK] in IJK frame.
15. Transform_4 Earth point EP to SC Range Vector[rvI, rvJ, rvK] in IJK frame To EP to SC Range Vector[rvS, rvE, rvZ] in SEZ frame.
16. Elevation(EL) and Azimuth(AZ) angle of Sun at Earth Observation point EP
17. Distance in km from Earth observation point(EP) to Sub Sun point(SS) and Earth Velocity meter per sec in orbit at time input UT.
18. Earth State Position Vector [X, Y, Z] in km at time input UT.
19. Earth State Velocity Vector [Vx, Vy, Vz] in meter per sec at time input UT.
20. Earth Orbit Normal Vector [Wx, Wy, Wz] in km and angles Delta, i, RA at time input UT; Normal is line perpendicular to orbit plane.
21. Transform Earth State Vectors To Earth position Keplerian elements.
22. Transform Earth position Keplerian elements To Earth State Vectors .

The values of all these parameters are Computed are at Standard Epoch JD2000 and when Earth is at Perihelion, Aphelion, Equinoxes, and Solstices.

The time at perihelion, aphelion, equinoxes, and solstices, were computed earlier for the input year .

Move on to Find Position of Earth on Celestial Sphere, at Seven different Time events, the Utilities of OM-MSS Software (Sections - 4.1 to 4.8).

- (a) Time Event - Standard Epoch JD2000 ;
- (b) Time Event - when Earth is at Perihelion ;
- (c) Time Event - when Earth is at Vernal equinox ;
- (d) Time Event - when Earth is at Summer solstice ;
- (e) Time Event - when Earth is at Aphelion ;
- (f) Time Event - when Earth is at Autumnal equinox ;
- (g) Time Event - when Earth is at Winter solstices ;

Move on to Compute Position of Earth on Celestial Sphere at all seven astronomical Time events in orbit, Earth around Sun.

Next Section - 4.1 Position of earth at standard epoch time JD2000

Earth Positional Parameters on Celestial Sphere : Input Time (UT) Standard Epoch JD2000

1. Find Position of Earth on Celestial Sphere at Input UT Standard Epoch time JD2000 .

Input UT Time, Standard Epoch time JD2000 : year = 2000, month = 1, day = 1, hour = 12, minute = 0, seconds = 0.00000
 Julian Day = 2451545.00000, year_day_decimal = 0.50000, day_hour_decimal = 12.00000
 Observation Point on Earth (Bhopal, India) : Lat +ve or -ve 0 to 90 deg = 23.25993 ie deg = 23, min = 15, sec = 35.76
 Log 0 to 360 deg = 77.41261 ie deg = 77, min = 24, sec = 45.41
 Alt from earth surface in km = 0.49470

First Compute the Sun Position on Celestial Sphere, then Compute the Earth Position on Celestial Sphere.

(A) Computed Values for SUN POSITION on Celestial Sphere at Input Ut Time : (Sr. No 1 - 22)

01. Earth around Sun Mean motion rev per day (mm)	= 0.0027377786	02. Semi-major axis in km considering oblateness (SMA)	= 149598616.31172
03. Earth mean motion deg per day using SMA (mm)	= 0.9856003000	04. Sun mean movement deg per day (n sun)	= 0.9856003000
05. Eccentricity of earth orbit (e sun)	= 0.0167102190	06. Perihelion to input time diff in Julian days	= -2.5081161195
07. Mean anomaly in deg per day from n_sun (m sun)	= -2.4719999999	08. Sun Mean Longitude in deg (Lmean)	= 280.4600000000
09. Earth Mean anomaly in deg (ME)	= 357.5280000000	10. Sun Ecliptic Longitude in deg (Lsun)	= 280.3756801972
11. Obliquity of ecliptic in deg (Epcylone)	= 23.4392794444	12. Sun Right ascension in deg (Alpha)	= 281.2858630915
13. Sun Declination in deg (Delta)	= -23.0337026521	14. Sun Mean distance in km (As)	= 149598616.31172
15. Sun Radial distance from earth in km (Rs)	= 147101227.61694	16. Sun Nodal elongation in deg (U sun)	= -79.6243198028
17. Sun Mean anomaly in deg (M sun)	= 357.5280000002	18. Sun Eccentric anomaly in deg (E sun)	= 357.4860040557
19. Sun True anomaly in deg (T sun)	= 357.4436516380	20. Sun Argument of perigee in deg (W sun)	= 282.9320285593
21. Sun True anomaly in deg from U & W (V sun)	= 357.4436516380	22. Sun Distance in km (d sun)	= 147101040.52850

Sun Ecliptic latitude is always nearly zero (the value never exceeds 0.00033 deg)

These Values are applied as input for Computing Earth Position on Celestial Sphere around Sun at same input UT Time.

(B) Computed Values for EARTH POSITION on Celestial Sphere around Sun at same Input Ut Time : (Sr. No 1 - 22)

Input Time year = 2000, month = 1, day = 1, hour = 12, minute = 0, seconds = 0.00000, corresponding Julian Day = 2451545.0000000000
 Observation Point on Earth : Lat +ve or -ve 0 to 90 deg = 23.25993, Log 0 to 360 deg = 77.41261, Alt from earth surface in km = 0.49470
 Sun position on Celestial sphere at input time, computed above total 22 parameters.

Output Earth Position on Celestial sphere around Sun : **Computed below around 120 parameters, presented in 1-22 groups.**

Number is large, because some parameters are computed using more than one model equation, that require different inputs.

This helps in validation of results and understanding the different input considerations.

01. Finding GST Greenwich sidereal time and GHA Greenwich hour angle in 0 to 360 deg, at input UT time YY MM DD HH.

Note - for GST, the year 1900 JAN day_1 hr 1200 is ref for time difference in terms of julian_century,

for GHA, the year 2000_JAN_day_1 hr_1200 is ref for time difference in terms of julian days.

Inputs : Time UT year = 2000, month = 1, day = 1, hour = 12, minute = 0, seconds = 0.00000

Outputs : GST & GHA in 0-360 deg over Greenwich.

E01A 011. GST Greenwich sidereal time in 0-360 deg, over Greenwich = 280.46030, hr = 18, min = 41, sec = 50.47200

E01B 011. GHA Greenwich hour angle in 0 to 360 deg, over Greenwich = 280.46743, deg = 280, min = 28, sec = 2.73693

02. Finding Earth latitude & longitude pointing to Sun Ecliptic longitude(Lsun).

Inputs : earth inclination, sun true anomaly T_Sun, sun argument of perigee W_Sun, sun right ascension Alpha,

earth equator radius, GST at input UT, log SS & EP, earth orbit radius EC to SC

Outputs : Earth lat & log pointing to Lsun.

E02A 011. Earth latitude +ve or -ve in 0 to 90 deg at UT time = -23.03 ie deg = -23, min = 2, sec = 1.33

E02B 011. Earth longitude 0 to 360 deg = 0.83 ie deg = 0, min = 49, sec = 32.03

03. Finding LST over three longitudes, Greenwich log, Sun mean log (Lmean), and Sun ecliptic log (Lsun) .

Note - for LST, used sidereal time at Greenwich GST and desired geographic longitude

Inputs : At Time input UT - GST, Log of Greenwich, sun mean log Lmean, Sun ecliptic log Lsun.

Outputs : LST over Greenwich, Lmean, Lsun .

E03A 011. LST Local sidereal time in 0-360 deg, over Greenwich longitude = 280.46030, hr = 18, min = 41, sec = 50.47200

E03B 011. LST Local sidereal time in 0-360 deg, over Sun mean longitude (Lmean) = 200.92030, hr = 13, min = 23, sec = 40.87200
 E03C 011. LST Local sidereal time in 0-360 deg, over Sun epliptic longitude (Lsun) = 200.83598, hr = 13, min = 23, sec = 20.63525

04. Finding ST0 sidereal time over Greenwich longitude = 0.0, at time input Year JAN day 1 hr 00.

Note - this is sidereal time ST at UT year, month = 1, day = 1, hours decimal = 0.0 and geographic longitude = 0.0

Inputs : Time input UT Year, JAN day 1 hr 00, Log 0.0

Outputs : ST0 over Greenwich

E04 011. ST0 Sidereal time in 0-360 deg, over Greenwich at input UT year, MM 1, DD 1, HH 00 = 99.96748, hr = 6, min = 39, sec = 52.19432

05. Finding ST sidereal time over three longitudes of, Greenwich log, Sun mean log (Lmean), and Sun epliptic log (Lsun) .

Note - this is local sidereal time LST; (LST = GST at UT time + geographic longitude).

Inputs : At Time input UT - Log 0.0, Log Lmean, Log Lsun

Outputs : ST over Greenwich, Lmean, Lsun.

E05A 011. ST Sidereal time in 0-360 deg, over Greenwich at input UT time = 280.46030, hr = 18, min = 41, sec = 50.47200

E05B 011. ST Sidereal time in 0-360 deg, over Sun mean longitude (Lmean) at input UT time = 200.92030, hr = 13, min = 23, sec = 40.87200

005C 011. ST Sidereal time in 0-360 deg, over Sun longitude (Lsun) at input UT time = 200.83598, hr = 13, min = 23, sec = 20.63525

06. Finding H hour angle in 0 to 360 deg over longitudes of, Greenwich, Lmean, Lsun, Earth Sub Sun point SS, Earth Obseration point EP.

Note - used Sun Right ascension Alpha at input time; (hour angle HA = LST - Alpha).

Inputs : At Time input UT - Sun Right ascension Alpha and ST Sidereal time over longitudes 0.0, Lmean, Lsun, SS, EP

Outputs : Hour Angles over Greenwich, Lmean, Lsun, SS, EP

E06A 011. H hour angle 0-360 deg, over Greenwich, = 359.17444, deg = 359, min = 10, sec = 27.97287

E06B 011. H hour angle 0-360 deg, over Lmean, = 279.63444, deg = 279, min = 38, sec = 3.97287

E06C 011. H hour angle 0-360 deg, over Lsun, = 279.55012, deg = 279, min = 33, sec = 0.42158

E06D 011. H hour angle 0-360 deg, over SS, = 0.00000, deg = 0, min = 0, sec = 0.00000

E06E 011. H hour angle 0-360 deg, over EP, = 76.58705, deg = 76, min = 35, sec = 13.38687

07. Finding Delta E is Equation of Time in seconds, at time input UT.

Note - this value in seconds accounts for relative movement of sun in elliptical orbit w.r.t earth and effect of obliquity of the ecliptic;

its maximum value is 16 minutes (960 sec.); Delta E is computed using time in days from the perihelion, n_sun_deg and w_sun at input UT.

Inputs : Time input UT in JD, time perihelion in JD, Sun mean movement n_sun, Eccentricity of earth orbit E_Sun

Outputs : Delta E time_equation in seconds.

E07 011. Delta E Time Equation in seconds = 191.54215, hr = 0, min = 3, sec = 11.54215

08. Finding GST Greenwich sidereal time, and GHA Greenwich hour angle 0 to 360 deg at time when earth is at perihelion.

Inputs : Time in JD when earth at perihelion YY = 2000, MM = 1, DD = 4, hr = 0, min = 11, sec = 41.23

Outputs : GST & GHA in 0-360 deg over Greenwich when earth is at perihelion

E08A 011. GST sidereal time in 0-360 deg over Greenwich at time when earth is at perihelion = 105.85422, hr = 7, min = 3, sec = 25.01305

E08B 011. GHA hour angle in 0-360 deg over Greenwich at time when earth is at perihelion = 105.85427, hr = 7, min = 3, sec = 25.02537

09. Finding ST sidereal time and MST mean sidereal time, over Greenwich, using Earth mean motion rev per day .

Inputs : GST when earth at perihelion, earth rotation rate, ref. JD2000, time input UT in JD, time perihelion in JD.

Outputs : STP, angle perihelion to input JD, ST over Greenwich, MSTO & MST over Greenwich, solar time

E09A 011. STP sidereal time in 0-360 deg over Greenwich when earth at perihelion = 105.85422, hr = 7, min = 3, sec = 25.01305

E09B 011. Angle in 0-360 deg from earth at perihelion to input JD using earth rotational rate = 174.60617,

E09C 011. ST in 0-360 deg over Greenwich using STP and angle from perihelion at input JD = 280.46039, hr = 18, min = 41, sec = 50.49311

E09D 011. ST in 0-360 deg over Greenwich using STP and earth rotation at UT time = 280.46039, hr = 18, min = 41, sec = 50.49311

E09E 011. MSTO in deg, over Greenwich using JD century days, ref J2000 to I/P YY, M1, D1 hr 00 = 99.96779, hr = 6, min = 39, sec = 52.27073

E09F 011. MST in deg, over Greenwich using JD century in days from ref J2000 to UT time Y M D H = 280.46062, hr = 18, min = 41, sec = 50.54841

E09G 011. Solar time over Greenwich in JD (GMT or input UT - 12 hr) = YY 2000, MM 1, DD 1, hr 0, min 0, sec 0.000, ie JD 2451544.50000

10. Finding Earth orbit radius using true anomaly, Sub Sun point (SS) on earth surface and related parameters .

(a) Finding Earth orbit radius using true anomaly.

Inputs : semi-major axis SMA, eccentricity of earth orbit e_sun, sun true anomaly T_Sun

Outputs : earth orbital radius EC to SC (earth center to sun center)

E10A 011. earth orbital radius EC to SC km using true anomaly at UT time = 147101196.6616485400

(b) Finding Sub Sun point (SS) over earth surface (Latitude, Longitude, & Latitude radius) pointing to Sun Ecliptic Log (Lsun), Sun height from earth surface over SS, and LST over SS log at time input UT.

Note - for SS Latitude, used earth inclination, sun true anomaly T_sun and sun argument of perigee w_sun.
for SS Longitude, used Sun right ascension Alpha and sidereal time at Greenwich GST.

Inputs : earth inclination, sun true anomaly T_Sun, sun argument of perigee W_Sun, sun right ascension Alpha, earth equator radius, GST at input UT, log SS & EP, earth orbit radius EC to SC

Outputs : SS point Latitude, Longitude, Latitude radius, LST & LMT over SS .

E10B 011. SS point Latitude +ve or -ve in 0 to 90 deg at UT time = -23.03 ie deg = -23, min = 2, sec = 1.33

E10C 011. SS point Longitude 0 to 360 deg = 0.83 ie deg = 0, min = 49, sec = 32.03

E10D 011. SS point Latitude radius km at UT time = 6374.8796719602

E10E 011. Sun height km from earth surface over SS at UT time = 147094821.7819765800

E10F 011. LST local sidereal time in 0-360 deg over SS log at UT time, (LST = GST + log east) = 281.286 ie hr = 18, min = 45, sec = 8.68356
LST local sidereal time and LMT local mean time with date adjusted to calendar YY MM DD and UT hr mm sec.

E10G 011. LST local sidereal time at Sub Sun point (SS) YY = 2000, MM = 1, DD = 1, hr = 18, min = 45, sec = 8.68

E10H 011. LMT local Mean time at Sub Sun point (SS) YY = 2000, MM = 1, DD = 1, hr = 12, min = 3, sec = 18.14

(c) Finding LST and LMT over Earth point(EP) where Observer is, at time input UT.

Inputs : EP point Latitude, Longitude ,

Outputs : LST & LMT over EP .

E10I 011. LST local sidereal time in 0-360 deg at EP log at UT time, (LST = GST + log east) = 357.873 ie hr = 23, min = 51, sec = 29.57601
LST and LMT with date adjusted to calendar YY MM DD and UT hr mm sec.

E10J 011. LST local sidereal time at Earth point (EP) YY = 2000, MM = 1, DD = 1, hr = 23, min = 51, sec = 29.58

E10K 011. LMT local Mean time at Earth point (EP) YY = 2000, MM = 1, DD = 1, hr = 17, min = 9, sec = 39.03

Finding Earth to Sun Position Vectors coordinate in PQW, IJK, SEZ frames and the Vector Coordinate Transforms.

First defined coordinate systems, PQW, IJK, SEZ, then computed Position & Velocity vectors in these three coordinate systems.

(a) **Perifocal Coordinate System (PQW)**, is Earth Centered Inertial coordinate frame defined in terms of Kepler Orbital Elements.

The system is fixed with time (inertial), pointing towards orbit periapsis;

the system's origin is Earth center (EC), and its fundamental plane is the orbit plane;

the P-vector axis directed from EC toward the periapsis of the elliptical orbit plane,

the Q-vector axis sweeps 90 deg from P axis in the direction of the orbit,

the W-vector axis directed from EC in a direction normal to orbit plane, forms a right-handed coordinate system.

(b) **Geocentric Coordinate System (IJK)**, is also an Earth Centered Inertial (ECI) frame, a Conventional Inertial System (CIS).

The system is fixed with time (inertial), pointing towards vernal equinox;

the system's origin is Earth center (EC), and its fundamental plane is the equator;

the I-vector is +X-axis directed towards the vernal equinox direction on J2000, Jan 1, hr 12.00 noon,

the J-vector is +Y-axis sweeps 90 deg to the east in the equatorial plane,

the K-vector is +Z-axis directed towards the North Pole.

(c) **Topocentric Horizon Coordinate System (SEZ)**, is Non-Inertial coordinate frame, known as Earth-Centered Earth-Fixed Coordinates (ECEF).

The system moves with earth, is not fixed with time (non-inertial), is for use by observers on the surface of earth;

the observer's surface forms the fundamental plane, is tangent to earth's surface

the S-vector is +ve horizontal -axis directed towards South,

the E-vector is +ve horizontal -axis directed towards East,

the Z-vector is +ve normal directed upwards on earth surface.

Note that axis Z not necessarily pass through earth center, so not used to define as radius vector.

11. Finding Earth center(EC) to Sun center(SC) Range Vector[r_p , r_q , r] from in PQW frame, perifocal coordinate system.

Inputs : Semi-major axis (SMA), Eccentricity of earth orbit (e_{sun}), Sun eccentric anomaly (E_{sun})

Outputs : Vector(r , r_p r_q) in PQW frame

E11A 011. r earth pos vector magnitude EC to SC km in PQW frame perifocal cord at UT time = 147101196.66165

E11B 011. rp earth pos vector component EC to SC km in PQW frame perifocal cord at UT time = 146954807.4835177700

E11C 011. rq earth pos vector component EC to SC km in PQW frame perifocal cord at UT time = -6560992.0569400741

Note - r earth pos vector magnitude EC to SC km in PQW frame is same as earth orbital radius computed before using true anomaly.

12. Transform_1 Earth position EC to SC Range Vector[rp, rq] in PQW frame To Range Vector[rI, rJ, rK] in IJK frame, inertial system cord.

Inputs : Vector(rp, rq) EC_to_SC km in frame PQW , Alpha rd, w_sun rd, earth_inclination rd ,

Outputs : Vector(rI, rJ, rK, r) EC_to_SC km in frame IJK

E12A 011. rI earth pos vector component EC to SC km frame IJK at UT time = -125003885.4456264400

E12B 011. rJ earth pos vector component EC to SC km frame IJK at UT time = -51961735.9769164550

E12C 011. rK earth pos vector component EC to SC km frame IJK at UT time = -57556656.2358423690

E12D 011. r earth pos vector magnitude EC to SC km frame IJK at UT time = 147101196.6616485400

Note - r earth pos vector magnitude EC to SC km in PQW frame is same as that computed above in PQW frame.

13. Transform_2 Earth point EP(lat, log, hgt) To EC to SC Range Vector[RI, RJ, RK, R] in IJK frame.

Inputs : earth equator radius_km, earth point EP(lat deg, log deg, hgt meter),

LST_local_sidereal_time_in_0_to_360_deg_at_EP_log_at_UT_time,

Outputs : Vector(RI, RJ, RK, R) Range EC to EP in IJK frame

E13A 011. RI pos vector component EC to EP km frame IJK at UT time = 5853.1109516962

E13B 011. RJ pos vector component EC to EP km frame IJK at UT time = -217.3619305363

E13C 011. RK pos vector component EC to EP km frame IJK at UT time = 2517.6312937817

E13D 011. R pos vector magnitude EC to EP km frame IJK at UT time = 6375.3134317570

14. Transform_3 Earth position EC to SC Range Vectors [rI rJ rK] & [RI RJ RK] To EP to SC Range Vector[rvI, rvJ, rvK] in IJK frame.

Inputs : Vector(rI rJ rK) position EC_to_SC km in frame IJK , Vector(RI RJ RK) range EC to SC km in IJK frame,

Outputs : Vector(rvI, rvJ, rvK, rv) range EP to SC in IJK frame

E14A 011. rvI range vector component EP to SC km frame IJK at UT time = -125009738.5565781300

E14B 011. rvJ range vector component EP to SC km frame IJK at UT time = -51961518.6149859200

E14C 011. rvK range vector component EP to SC km frame IJK at UT time = -57559173.8671361510

E14D 011. rv range vector magnitude EP to SC km frame IJK at UT time = 147107078.8474394100

15. Transform_4 Earth point EP to SC Range Vector[rvI, rvJ, rvK] in IJK frame To EP to SC Range Vector[rvS, rvE, rvZ] in SEZ frame.

Inputs : lat_pos_neg_0_to_90_deg_at_EP_at_time_UT , LST_local_sidereal_time_in_0_to_360_deg_at_EP_log_at_UT_time

Vector(rvI, rvJ, rvK, rv) range EP to SC km in IJK frame,

Outputs : Vector(rvS, rvE, rvZ, rv) range EP to SC km in SEZ frame

E15A 011. rvS range vector component EP to SC km frame SEZ at UT time = 4309690.2262858748

E15B 011. rvE range vector component EP to SC km frame SEZ at UT time = -56564906.7140329630

E15C 011. rvZ range vector component EP to SC km frame SEZ at UT time = -135728886.1871818300

E15D 011. rv range vector magnitude EP to SC km frame SEZ at UT time = 147107078.8474394100

16. Finding Elevation(EL) and Azimuth(AZ) angle of Sun at Earth Observation point EP .

Note : Results **computed using 4 different formulations**, each require different inputs to give EL & AZ angles.

For all situations of Object and Observer positions, a combination of Latitude N/S & Longitude E/W :

Method 1 : for both EL & AZ angles, this does not provide correct results ;

Method 2 : for only EL angle, this provides consistent, unambiguous correct results.

but for AZ angles the results are ambiguous, need corrections by adding or subtracting values as 180 or 360 or sign change.

Method 3 : same as method 2, for EL angle, the results are correct, but for AZ angles the results are ambiguous, need corrections.

Method 4 : for finding Azimuth and Distance but not for finding Elevation angle;

for AZ angles, this provides correct unambiguous results that need no further corrections.

Therefore for Elevation (EL) angle Method 3 results are accepted and for Azimuth (AZ) angle Method 4 results are accepted .

Results verified from other sources; Ref URLs <http://www.ga.gov.au/geodesy/astro/smpos.jsp#intzone> .

NOAA Research <http://www.esrl.noaa.gov/gmd/grad/solcalc/> , and <http://aa.usno.navy.mil/data/docs/AltAz.php>

Xavier Jubier, Member IAU http://xjubier.free.fr/en/site_pages/astrometry/ephemerides.html

Rem: SS point lat deg = -23.03, log deg = 0.83 YY = 2000, MM = 1, DD = 1, hr = 12, min = 3, sec = 18.14

EP point lat deg = 23.26, log deg = 77.41 YY = 2000, MM = 1, DD = 1, hr = 17, min = 9, sec = 39.03

Elevation(EL) & Azimuth(AZ) angle of Sun at Earth Observation point EP, (**method 1** - computed values may be Ambiguous or Incorrect).

Inputs : Vector[rvS, rvE, rvZ] range EP to SC km in SEZ frame

Outputs : Elevation(EL) & Azimuth(AZ) of Sun at EP

E16A 011. Elevation angle deg of Sun at EP using rv SEZ at UT time = -67.3171664896

E16B 011. Azimuth angle deg of sun at EP using rv SEZ at UT time = 265.6430420660

Elevation(EL) & Azimuth(AZ) angle of Sun at Earth Observation point EP, (method 2 - computed AZ values may be ambiguous & incorrect).

Inputs : Time input UT YY MM DD HH, Equator radius, EP lat & log, SS lat & log, Sun declination Delta

Outputs : Elevation(EL) & Azimuth(AZ) of Sun at EP

E16C 011. Elevation angle deg of Sun at EP using Sun declination diff log range EP to SC = 2.38469

E16D 011. Azimuth angle deg of sun at EP using sun declination diff log range EP to SC = -153.63057

Elevation(EL) & Azimuth(AZ) angle of Sun at Earth Observation point EP, (method 3 - computed AZ values may be ambiguous & incorrect).

Inputs : Time input UT YY MM DD HH, Equator radius, EP lat & log, SS lat & log, Sun hgt from EC, Sun range from EP

(Sun hgt from EC = earth orbit radius EC to SC km ; Sun range from EP = rv range vector EP to SC km frame SEZ)

Outputs : Elevation(EL) & Azimuth(AZ) of Sun at EP

E16E 011. Elevation angle deg of Sun at EP using Sun hgt diff log range EP to SC = 2.38221 ie deg = 2, min = 22, sec = 55.96

E16F 011. Azimuth angle deg of sun at EP using sun hgt diff log range EP to SC = 243.63057 ie deg = 243, min = 37, sec = 50.05

Azimuth(AZ) angle of Sun at Earth Observation point EP, (method 4 - computed AZ values is unambiguous & correct).

Inputs : Time input UT YY MM DD HH, EP lat & log, SS lat & log

Outputs : Azimuth(AZ) of Sun at EP

E16G 011. Azimuth angle deg of sun at EP using sun hgt diff log range EP to SC = 243.63057 ie deg = 243, min = 37, sec = 50.05

Due to such incorrect results, finally for Elevation (EL) Method 3 results and for Azimuth (AZ) Method 4 results are accepted.

Finally accepted Elevation angle deg of Sun from EP to SC = 2.3822113375 ie deg = 2, min = 22, sec = 55.96

Finally accepted Azimuth angle_deg of Sun from EP to SC = 243.6305706573 ie deg = 243, min = 37, sec = 50.05

Distance in km from Earth observation point(EP) to Sub Sun point(SS) and Earth Velocity meter per sec in orbit at time input UT.

17. Finding Distance in km from Earth observation point(EP) to Sub Sun point(SS) over Earth surface .

Inputs : EP lat & log, SS lat & log,

Outputs : Distance in km from EP to SS over Earth surface

E17A 011. Distance in km Earth observation point(EP) to Sub Sun point(SS) = 9753.27897

Finding Earth Velocity meter per sec in orbit in frame PQW

Inputs : semi-major axis SMA, GM_Sun, earth pos r EC to SC frame IJK, eccentricity of earth orbit e_Sun, sun eccentric anomaly E_Sun

Outputs : Earth Velocity magnitude and component Xw Yw in frame PQW in meter per sec

E17B 011. Velocity magnitude meter per sec using GM, SMA, r earth EC to SC frame IJK at UT time = 30286.0666340612

E17C 011. Velocity component meter per sec in orbit Xw using GM, e_Sun, SMA, E_Sun at UT time = 1328.6358841510

E17D 011. Velocity component meter per sec in orbit Yw using GM, e_Sun, SMA, E_Sun at UT time = 30256.9092745796

Finding Earth Velocity Vector [vX, vY, vZ] in meter per sec in orbit; a Transform of [Xw, Yw] in frame PQW To [vX, vY, vZ] in frame XYZ

Inputs : velocity component (Xw, Yw), sun right ascension Alpha, Sun Argument of perigee W_Sun, inclination Epcylone

Outputs : earth velocity vector(vX, vY, vZ, vR) meter per sec in frame XYZ

E17E 011. vX earth Velocity vector meter per sec using Xw Yw frame PQW RA w i at UT time = 10756.7530226526

E17F 011. vY earth Velocity vector meter per sec using Xw Yw frame PQW RA w i at UT time = -28227.5170485498

E17G 011. vZ earth Velocity vector meter per sec using Xw Yw frame PQW RA w i at UT time = 2178.3888188937

E17H 011. vR earth Velocity magnitude meter per sec using Xw Yw frame PQW RA w i at UT time = 30286.0666340612

Earth State Vectors : Position [X, Y, Z] in km and Velocity [Vx, Vy, Vz] in meter per sec, at time input UT.

18. Finding Earth State Position Vector [X, Y, Z] in km at time input UT.

Inputs : position vector(r_I, r_J, r_K, r) in frame IJK values assigned to state position vector

Outputs : State Position Vector(X, Y, Z, R) in km, frame XYZ

E18A 011. State vector position X km at UT time = -125003885.4456264400

E18B 011. State vector position Y km at UT time = -51961735.9769164550

E18C 011. State vector position Z km at UT time = -57556656.2358423690

E18D 011. State vector position R km at UT time = 147101196.6616485400

19. Finding Earth State Velocity Vector [Vx, Vy, Vz] in meter per sec at time input UT.

Inputs : velocity vector(v_X, v_Y, v_Z, v_R) meter per sec in frame XYZ values assigned to state velocity vector

Outputs : state velocity vector(Vx, Vy, Vz, V) meter per sec, frame XYZ

E19A 011. State vector velocity Vx meter per sec at UT time = 10756.7530226526

E19B 011. State vector velocity Vy meter per sec at UT time = -28227.5170485498

E19C 011. State vector velocity Vz meter per sec at UT time = 2178.3888188937

019D 011. State vector velocity V meter per sec at UT time = 30286.0666340612

20. Earth Orbit Normal Vector [Wx, Wy, Wz] in km and angles Delta, i, RA at time input UT; Normal is line perpendicular to orbit plane.

Inputs : earth pos r EC to SC frame IJK, inclination Epcylone, sun right ascension Alpha

Outputs : earth orbit normal vector (Wx, Wy, Wz, W) in km

E20A 011. Earth orbit normal W km using r earth pos frame IJK inclination Alpha = 147101196.6616485400

E20B 011. Earth orbit normal Wx km using r earth pos frame IJK inclination Alpha = -57381991.7843673230

E20C 011. Earth orbit normal Wy km using r earth pos frame IJK inclination Alpha = -11451330.6380786910

E20D 011. Earth orbit normal Wz km using r earth pos frame IJK inclination Alpha = 134962721.1668659400

020E 011. Earth orbit normal Delta W deg using r earth pos frame IJK inclination Alpha = 66.5607205617

E20F 011. Earth orbit normal Inclination i deg using normal_Delta_W = 23.4392794383

E20G 011. Earth orbit normal Alpha W deg using r earth pos frame IJK, inclination, Alpha = 11.2858630915

E20H 011. Earth orbit normal Right ascension of ascending node using normal Alpha, W = 101.2858630915

Transform Earth State Vectors to Earth position Keplerian elements.

21. Finding Earth position Keplerian elements computed using State Vector, at time input UT.

Inputs : State vector year, days decimal of year, revolution, node, State Position Vector [X, Y, Z], State Velocity Vector [Vx, Vy, Vz]

Outputs : Keplerian elements : year, days decimal of year, revolution, node, inclination, right ascension, eccentricity, argument of perigee, mean anomaly, mean motion rev per day, mean angular velocity rev per day, mean motion rev_per_day from SMA considering oblateness

E21A 011. Keplerian elements year = 2000, days_decimal_of_year = 0.50000, revolution no = 1, node = 2 ie decending

E21B 011. inclination_deg = 23.4392794383

E21C 011. right ascension ascending node deg = 281.2858630915

E21D 011. eccentricity = 0.0167102190

E21E 011. argument of perigee_deg = 282.9320285591

E21F 011. mean anomaly deg = 357.5280000003

E21G 011. mean_motion rev per day = 0.0027377786

E21H 011. mean angular velocity rev_per_day = 0.0027377786

E21I 011. mean motion rev per day using SMA considering oblateness = 0.0027377786

Transform Earth position Keplerian elements to Earth State Vectors .

22. Finding Earth position State Vectors, computed using Keplerian elements at time input UT

(computed again to validate model equations, Keplerian elements to State Vectors & back)

Inputs : Keplerian elements : year, days decimal of year, revolution, node, inclination, right ascension, eccentricity, argument of perigee, mean anomaly, mean motion rev per day, mean angular velocity rev per day, mean motion rev_per_day from SMA considering oblateness

Outputs : State vector year, days decimal of year, revolution, node, State Position Vector [X, Y, Z], State Velocity Vector [Vx, Vy, Vz]

E22A 011. State vectors year = 2000, days_decimal_of_year = 0.50000, revolution no = 1, node = 2 ie decending

E22B 011. state vector position X km = -125003885.4456292700, state vector velocity Vx meter per sec = 10756.7530226511

E22C 011. state vector position Y km = -51961735.9769086610, state vector velocity Vy meter per sec = -28227.5170485504

E22D 011. state vector position Z km = -57556656.2358429060, state vector velocity Vz meter per sec = 2178.3888188931

E22E 011. state vector position R km = 147101196.6616483900, state vector velocity V meter per sec = 30286.0666340612

Note : Computation of all above parameters, grouped in 1 to 22, corresponds to time

(a) Universal time over Greenwich (UT/GMT) : Year = 2000, Month = 1, Day = 1, Hour = 12, Min = 0, Sec = 0.000

(b) Mean Solar time (MST) over Earth Observation point (EP) : Year = 2000, Month = 1, Day = 1, Hour = 17, Min = 9, Sec = 39.028

Move on to next Astronomical event in orbit Earth around Sun.

Next Section - 4.2 Position of earth at time when earth is at perihelion

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Earth Positional Parameters on Celestial Sphere : Input Year Time when Earth is at Perihelion.

2. Finding Position of Earth on Celestial Sphere at Input Universal Time, when Earth is at Perihelion .

Input UT Time, when Earth is at Perihelion : year = 2013, month = 1, day = 3, hour = 9, minute = 11, seconds = 56.61639
 Julian Day = 2456295.88329, year_day_decimal = 2.38329, day_hour_decimal = 9.19906
Observation Point on Earth (Bhopal, India) : Lat +ve or -ve 0 to 90 deg = 23.25993 ie deg = 23, min = 15, sec = 35.76
 Log 0 to 360 deg = 77.41261 ie deg = 77, min = 24, sec = 45.41
 Alt from earth surface in km = 0.49470

First Compute the **Sun Position** on Celestial Sphere, then Compute the **Earth Position** on Celestial Sphere.

(A) Computed Values for SUN POSITION on Celestial Sphere at Input Ut Time : (Sr. No 1 - 22)

01. Earth around Sun Mean motion rev per day (mm)	=	0.0027377786	02. Semi-major axis in km considering oblateness (SMA)	=	149598616.31172
03. Earth mean motion deg per day using SMA (mm)	=	0.9856003000	04. Sun mean movement deg per day (n sun)	=	0.9856003000
05. Eccentricity of earth orbit (e sun)	=	0.0167102190	06. Perihelion to input time diff in Julian days	=	0.0000000000
07. Mean anomaly in deg per day from n_sun (m sun)	=	0.0000000000	08. Sun Mean Longitude in deg (Lmean)	=	283.1557666033
09. Earth Mean anomaly in deg (ME)	=	0.0000000001	10. Sun Ecliptic Longitude in deg (Lsun)	=	283.1557666033
11. Obliquity of ecliptic in deg (Epcylone)	=	23.4375874462	12. Sun Right ascension in deg (Alpha)	=	284.2922173002
13. Sun Declination in deg (Delta)	=	-22.7872783368	14. Sun Mean distance in km (As)	=	149598616.31172
15. Sun Radial distance from earth in km (Rs)	=	147098823.43315	16. Sun Nodal elongation in deg (U sun)	=	-76.8442333967
17. Sun Mean anomaly in deg (M sun)	=	0.0000000000	18. Sun Eccentric anomaly in deg (E sun)	=	0.0000000000
19. Sun True anomaly in deg (T sun)	=	0.0000000000	20. Sun Argument of perigee in deg (W sun)	=	283.1557666033
21. Sun True anomaly in deg from U & W (V sun)	=	0.0000000000	22. Sun Distance in km (d sun)	=	147098790.67105

Sun Ecliptic latitude is always nearly zero (the value never exceeds 0.00033 deg)

These Values are applied as input for Computing Earth Position on Celestial Sphere around Sun at same input UT Time.

(B) Computed Values for EARTH POSITION on Celestial Sphere around Sun at same Input Ut Time : (Sr. No 1 - 22)

Input Time year = 2013, month = 1, day = 3, hour = 9, minute = 11, seconds = 56.61639, corresponding Julian Day = 2456295.8832941712
 Observation Point on Earth : Lat +ve or -ve 0 to 90 deg = 23.25993, Log 0 to 360 deg = 77.41261, Alt from earth surface in km = 0.49470
 Sun position on Celestial sphere at input time, computed above total 22 parameters.

Output Earth Position on Celestial sphere around Sun : **Computed below around 120 parameters, presented in 1-22 groups.**
 Number is large, because some parameters are computed using more than one model equation, that require different inputs.
 This helps in validation of results and understanding the different input considerations.

01. Finding GST Greenwich sidereal time and GHA Greenwich hour angle in 0 to 360 deg, at input UT time YY MM DD HH.

Note - for GST, the year 1900 JAN day_1 hr 1200 is ref for time difference in terms of julian_century,
 for GHA, the year 2000_JAN_day_1 hr_1200 is ref for time difference in terms of julian days.

Inputs : Time UT year = 2013, month = 1, day = 3, hour = 9, minute = 11, seconds = 56.61639

Outputs : GST & GHA in 0-360 deg over Greenwich.

E01A 011. GST Greenwich sidereal time in 0-360 deg, over Greenwich = 241.14177, hr = 16, min = 4, sec = 34.02452
 E01B 011. GHA Greenwich hour angle in 0 to 360 deg, over Greenwich = 241.14717, deg = 241, min = 8, sec = 49.80962

02. Finding Earth Latitude & Longitude pointing to Sun Ecliptic Longitude(Lsun).

Inputs : earth inclination, sun true anomaly T_Sun, sun argument of perigee W_Sun, sun right ascension Alpha,
 earth equator radius, GST at input UT, log SS & EP, earth orbit radius EC to SC

Outputs : Earth Lat & Log pointing to Lsun.

E02A 011. Earth Latitude +ve or -ve in 0 to 90 deg at UT time = -22.79 ie deg = -22, min = 47, sec = 20.10
 E02B 011. Earth Longitude 0 to 360 deg = 43.15 ie deg = 43, min = 9, sec = 1.61

03. Finding LST over three longitudes, Greenwich log, Sun mean log (Lmean), and Sun ecliptic log (Lsun) .

Note - for LST, used sidereal time at Greenwich GST and desired geographic longitude

Inputs : At Time input UT - GST, Log of Greenwich, sun mean log Lmean, Sun ecliptic log Lsun.

Outputs : LST over Greenwich, Lmean, Lsun .

E03A 011. LST Local sidereal time in 0-360 deg, over Greenwich longitude = 241.14177, hr = 16, min = 4, sec = 34.02452

E03B 011. LST Local sidereal time in 0-360 deg, over Sun mean longitude (Lmean) = 164.29754, hr = 10, min = 57, sec = 11.40850
 E03C 011. LST Local sidereal time in 0-360 deg, over Sun epliptic longitude (Lsun) = 164.29754, hr = 10, min = 57, sec = 11.40850

04. Finding ST0 sidereal time over Greenwich longitude = 0.0, at time input Year JAN day 1 hr 00.

Note - this is sidereal time ST at UT year, month = 1, day = 1, hours decimal = 0.0 and geogrphic longitude = 0.0

Inputs : Time input UT Year, JAN day 1 hr 00, Log 0.0

Outputs : ST0 over Greenwich

E04 011. ST0 Sidereal time in 0-360 deg, over Greenwich at input UT year, MM 1, DD 1, HH 00 = 100.80678, hr = 6, min = 43, sec = 13.62710

05. Finding ST sidereal time over three longitudes of, Greenwich log, Sun mean log (Lmean), and Sun epliptic log (Lsun) .

Note - this is local sidereal time LST; (LST = GST at UT time + geogrphic longitude).

Inputs : At Time input UT - Log 0.0, Log Lmean, Log Lsun

Outputs : ST over Greenwich, Lmean, Lsun.

E05A 011. ST Sidereal time in 0-360 deg, over Greenwich at input UT time = 241.14177, hr = 16, min = 4, sec = 34.02452

E05B 011. ST Sidereal time in 0-360 deg, over Sun mean longitude (Lmean) at input UT time = 164.29754, hr = 10, min = 57, sec = 11.40850

005C 011. ST Sidereal time in 0-360 deg, over Sun longitude (Lsun) at input UT time = 164.29754, hr = 10, min = 57, sec = 11.40850

06. Finding H hour angle in 0 to 360 deg over longitudes of, Greenwich, Lmean, Lsun, Earth Sub Sun point SS, Earth Obseration point EP.

Note - used Sun Right ascension Alpha at input time; (hour angle HA = LST - Alpha).

Inputs : At Time input UT - Sun Right ascension Alpha and ST Sidereal time over longitudes 0.0, Lmean, Lsun, SS, EP

Outputs : Hour Angles over Greenwich, Lmean, Lsun, SS, EP

E06A 011. H hour angle 0-360 deg, over Greenwich, = 316.84955, deg = 316, min = 50, sec = 58.38552

E06B 011. H hour angle 0-360 deg, over Lmean, = 240.00532, deg = 240, min = 0, sec = 19.14529

E06C 011. H hour angle 0-360 deg, over Lsun, = 240.00532, deg = 240, min = 0, sec = 19.14529

E06D 011. H hour angle 0-360 deg, over SS, = 0.00000, deg = 0, min = 0, sec = 0.00000

E06E 011. H hour angle 0-360 deg, over EP, = 34.26217, deg = 34, min = 15, sec = 43.79952

07. Finding Delta E is Equation of Time in seconds, at time input UT.

Note - this value in seconds accounts for relative movement of sun in elliptical orbit w.r.t earth and effect of obliquity of the ecliptic;

its maximum value is 16 minutes (960 sec.); Delta E is computed using time in days from the perihelion, n_sun_deg and w_sun at input UT.

Inputs : Time input UT in JD, time perihelion in JD, Sun mean movement n_sun, Eccentricity of earth orbit E_Sun

Outputs : Delta E time_equation in seconds.

E07 011. Delta E Time Equation in seconds = 262.40497, hr = 0, min = 4, sec = 22.40497

08. Finding GST Greenwich sidereal time, and GHA Greenwich hour angle 0 to 360 deg at time when earth is at perihelion.

Inputs : Time in JD when earth at perihelion YY = 2013, MM = 1, DD = 3, hr = 9, min = 11, sec = 56.62

Outputs : GST & GHA in 0-360 deg over Greenwich when earth is at perihelion

E08A 011. GST sidereal time in 0-360 deg over Greenwich at time when earth is at perihelion = 241.14177, hr = 16, min = 4, sec = 34.02452

E08B 011. GHA hour angle in 0-360 deg over Greenwich at time when earth is at perihelion = 241.14717, hr = 16, min = 4, sec = 35.32064

09. Finding ST sidereal time and MST mean sidereal time, over Greenwich, using Earth mean motion rev per day .

Inputs : GST when earth at perihelion, earth rotation rate, ref. JD2000, time input UT in JD, time perihelion in JD.

Outputs : STP, angle perihelion to input JD, ST over Greenwich, MSTO & MST over Greenwich, solar time

E09A 011. STP sidereal time in 0-360 deg over Greenwich when earth at perihelion = 241.14177, hr = 16, min = 4, sec = 34.02452

E09B 011. Angle in 0-360 deg from earth at perihelion to input JD using earth rotational rate = 0.00000,

E09C 011. ST in 0-360 deg over Greenwich using STP and angle from perihelion at input JD = 241.14177, hr = 16, min = 4, sec = 34.02452

E09D 011. ST in 0-360 deg over Greenwich using STP and earth rotation at UT time = 241.14177, hr = 16, min = 4, sec = 34.02452

E09E 011. MSTO in deg, over Greenwich using JD century days, ref J2000 to I/P YY, M1, D1 hr 00 = 100.80714, hr = 6, min = 43, sec = 13.71450

E09F 011. MST in deg, over Greenwich using JD century in days from ref J2000 to UT time Y M D H = 241.14213, hr = 16, min = 4, sec = 34.11192

E09G 011. Solar time over Greenwich in JD (GMT or input UT - 12 hr) = YY 2013, MM 1, DD 2, hr 21, min 11, sec 56.616, ie JD 2456295.38329

10. Finding Earth orbit radius using true anomaly, Sub Sun point (SS) on earth surface and related parameters .

(a) Finding Earth orbit radius using true anomaly.

Inputs : semi-major axis SMA, eccentricity of earth orbit e_sun, sun true anomaly T_Sun

Outputs : earth orbital radius EC to SC (earth center to sun center)

E10A 011. earth orbital radius EC to SC km using true anomaly at UT time = 147098790.6710525200

(b) Finding Sub Sun point (SS) over earth surface (Latitude, Longitude, & Latitude radius) pointing to Sun Ecliptic Log (Lsun), Sun height from earth surface over SS, and LST over SS log at time input UT.

Note - for SS Latitude, used earth inclination, sun true anomaly T_sun and sun argument of perigee w_sun.
for SS Longitude, used Sun right ascension Alpha and sidereal time at Greenwich GST.

Inputs : earth inclination, sun true anomaly T_Sun, sun argument of perigee W_Sun, sun right ascension Alpha, earth equator radius, GST at input UT, log SS & EP, earth orbit radius EC to SC

Outputs : SS point Latitude, Longitude, Latitude radius, LST & LMT over SS .

E10B 011. SS point Latitude +ve or -ve in 0 to 90 deg at UT time = -22.79 ie deg = -22, min = 47, sec = 20.10

E10C 011. SS point Longitude 0 to 360 deg = 43.15 ie deg = 43, min = 9, sec = 1.61

E10D 011. SS point Latitude radius km at UT time = 6374.9450864748

E10E 011. Sun height km from earth surface over SS at UT time = 147092415.7259660400

E10F 011. LST local sidereal time in 0-360 deg over SS log at UT time, (LST = GST + log east) = 284.293 ie hr = 18, min = 57, sec = 10.21954
LST local sidereal time and LMT local mean time with date adjusted to calendar YY MM DD and UT hr mm sec.

E10G 011. LST local sidereal time at Sub Sun point (SS) YY = 2013, MM = 1, DD = 3, hr = 18, min = 57, sec = 10.22

E10H 011. LMT local Mean time at Sub Sun point (SS) YY = 2013, MM = 1, DD = 3, hr = 12, min = 4, sec = 32.72

(c) Finding LST and LMT over Earth point(EP) where Observer is, at time input UT.

Inputs : EP point Latitude, Longitude

Outputs : LST & LMT over EP .

E10I 011. LST local sidereal time in 0-360 deg at EP log at UT time, (LST = GST + log east) = 318.555 ie hr = 21, min = 14, sec = 13.13953
LST and LMT with date adjusted to calendar YY MM DD and UT hr mm sec.

E10J 011. LST local sidereal time at Earth point (EP) YY = 2013, MM = 1, DD = 3, hr = 21, min = 14, sec = 13.14

E10K 011. LMT local Mean time at Earth point (EP) YY = 2013, MM = 1, DD = 3, hr = 14, min = 21, sec = 35.64

Finding Earth to Sun Position Vectors coordinate in PQW, IJK, SEZ frames and the Vector Coordinate Transforms.

First defined coordinate systems, PQW, IJK, SEZ, then computed Position & Velocity vectors in these three coordinate systems.

(a) **Perifocal Coordinate System (POW)**, is Earth Centered Inertial coordinate frame defined in terms of Kepler Orbital Elements.

The system is fixed with time (inertial), pointing towards orbit periapsis;

the system's origin is Earth center (EC), and its fundamental plane is the orbit plane;

the P-vector axis directed from EC toward the periapsis of the elliptical orbit plane,

the Q-vector axis sweeps 90 deg from P axis in the direction of the orbit,

the W-vector axis directed from EC in a direction normal to orbit plane, forms a right-handed coordinate system.

(b) **Geocentric Coordinate System (IJK)**, is also an Earth Centered Inertial (ECI) frame, a Conventional Inertial System (CIS).

The system is fixed with time (inertial), pointing towards vernal equinox;

the system's origin is Earth center (EC), and its fundamental plane is the equator;

the I-vector is +X-axis directed towards the vernal equinox direction on J2000, Jan 1, hr 12.00 noon,

the J-vector is +Y-axis sweeps 90 deg to the east in the equatorial plane,

the K-vector is +Z-axis directed towards the North Pole.

(c) **Topocentric Horizon Coordinate System (SEZ)**, is Non-Inertial coordinate frame, known as Earth-Centered Earth-Fixed Coordinates (ECEF).

The system moves with earth, is not fixed with time (non-inertial), is for use by observers on the surface of earth;

the observer's surface forms the fundamental plane, is tangent to earth's surface

the S-vector is +ve horizontal-axis directed towards South,

the E-vector is +ve horizontal-axis directed towards East,

the Z-vector is +ve normal directed upwards on earth surface.

Note that axis Z not necessarily pass through earth center, so not used to define as radius vector.

11. Finding Earth center(EC) to Sun center(SC) Range Vector[r_p , r_q , r] from in PQW frame, perifocal coordinate system.

Inputs : Semi-major axis (SMA), Eccentricity of earth orbit (e_{sun}), Sun eccentric anomaly (E_{sun})

Outputs : Vector(r , r_p r_q) in PQW frame

E11A 011. r earth pos vector magnitude EC to SC km in PQW frame perifocal cord at UT time = 147098790.67105

E11B 011. rp earth pos vector component EC to SC km in PQW frame perifocal cord at UT time = 147098790.6710524900

E11C 011. rq earth pos vector component EC to SC km in PQW frame perifocal cord at UT time = 0.0000000294

Note - r earth pos vector magnitude EC to SC km in PQW frame is same as earth orbital radius computed before using true anomaly.

12. Transform_1 Earth position EC to SC Range Vector[r_p , r_q] in PQW frame To Range Vector[r_I , r_J , r_K] in IJK frame, inertial system cord.

Inputs : Vector(r_p , r_q) EC_to_SC km in frame PQW , Alpha rd, w_{sun} rd, earth_inclination rd ,

Outputs : Vector(r_I , r_J , r_K , r) EC_to_SC km in frame IJK

E12A 011. r_I earth pos vector component EC to SC km frame IJK at UT time = -119085952.6308768100

E12B 011. r_J earth pos vector component EC to SC km frame IJK at UT time = -64886279.6011658240

E12C 011. r_K earth pos vector component EC to SC km frame IJK at UT time = -56976844.6160544460

E12D 011. r earth pos vector magnitude EC to SC km frame IJK at UT time = 147098790.6710524900

Note - r earth pos vector magnitude EC to SC km in PQW frame is same as that computed above in PQW frame.

13. Transform_2 Earth point EP(lat, log, hgt) To EC to SC Range Vector[R_I , R_J , R_K , R] in IJK frame.

Inputs : earth equator radius_km, earth point EP(lat deg, log deg, hgt meter),

LST_local_sidereal_time_in_0_to_360_deg_at_EP_log_at_UT_time,

Outputs : Vector(R_I , R_J , R_K , R) Range EC to EP in IJK frame

E13A 011. R_I pos vector component EC to EP km frame IJK at UT time = 4390.4491505062

E13B 011. R_J pos vector component EC to EP km frame IJK at UT time = -3876.8686176516

E13C 011. R_K pos vector component EC to EP km frame IJK at UT time = 2517.6312937817

E13D 011. R pos vector magnitude EC to EP km frame IJK at UT time = 6375.3134317570

14. Transform_3 Earth position EC to SC Range Vectors [r_I r_J r_K] & [R_I R_J R_K] To EP to SC Range Vector[rv_I , rv_J , rv_K] in IJK frame.

Inputs : Vector(r_I r_J r_K) position EC_to_SC km in frame IJK , Vector(R_I R_J R_K) range EC to SC km in IJK frame,

Outputs : Vector(rv_I , rv_J , rv_K , rv) range EP to SC in IJK frame

E14A 011. rv_I range vector component EP to SC km frame IJK at UT time = -119090343.0800273100

E14B 011. rv_J range vector component EP to SC km frame IJK at UT time = -64882402.7325481700

E14C 011. rv_K range vector component EP to SC km frame IJK at UT time = -56979362.2473482270

E14D 011. rv range vector magnitude EP to SC km frame IJK at UT time = 147101610.1930285400

15. Transform_4 Earth point EP to SC Range Vector[rvI, rvJ, rvK] in IJK frame To EP to SC Range Vector[rvS, rvE, rvZ] in SEZ frame.

Inputs : lat_pos_neg_0_to_90_deg_at_EP_at_time_UT , LST_local_sidereal_time_in_0_to_360_deg_at_EP_log_at_UT_time

Vector(rvI, rvJ, rvK, rv) range EP to SC km in IJK frame,

Outputs : Vector(rvS, rvE, rvZ, rv) range EP to SC km in SEZ frame

E15A 011. rvS range vector component EP to SC km frame SEZ at UT time = 34055206.8052048240

E15B 011. rvE range vector component EP to SC km frame SEZ at UT time = -127461490.7613908900

E15C 011. rvZ range vector component EP to SC km frame SEZ at UT time = -65059165.2553297430

E15D 011. rv range vector magnitude EP to SC km frame SEZ at UT time = 147101610.1930285100

16. Finding Elevation(EL) and Azimuth(AZ) angle of Sun at Earth Observation point EP .

Note : Results **computed using 4 different formulations**, each require different inputs to give EL & AZ angles.

For all situations of Object and Observer positions, a combination of Latitude N/S & Longitude E/W :

Method 1 : for both EL & AZ angles, this does not provide correct results ;

Method 2 : for only EL angle, this provides consistent, unambiguous correct results.

but for AZ angles the results are ambiguous, need corrections by adding or subtracting values as 180 or 360 or sign change.

Method 3 : same as method 2, for EL angle, the results are correct, but for AZ angles the results are ambiguous, need corrections.

Method 4 : for finding Azimuth and Distance but not for finding Elevation angle;

for AZ angles, this provides correct unambiguous results that need no further corrections.

Therefore for Elevation (EL) angle Method 3 results are accepted and for Azimuth (AZ) angle Method 4 results are accepted .

Results verified from other sources; Ref URLs <http://www.ga.gov.au/geodesy/astro/smpos.jsp#intzone> .

NOAA Research <http://www.esrl.noaa.gov/gmd/grad/solcalc/> , and <http://aa.usno.navy.mil/data/docs/AltAz.php>

Xavier Jubier, Member IAU http://xjubier.free.fr/en/site_pages/astrometry/ephemerides.html

Rem: SS point lat deg = -22.79, log deg = 43.15 YY = 2013, MM = 1, DD = 3, hr = 12, min = 4, sec = 32.72

EP point lat deg = 23.26, log deg = 77.41 YY = 2013, MM = 1, DD = 3, hr = 14, min = 21, sec = 35.64

Elevation(EL) & Azimuth(AZ) angle of Sun at Earth Observation point EP, (**method 1** - computed values may be Ambiguous or Incorrect).

Inputs : Vector[rvS, rvE, rvZ] range EP to SC km in SEZ frame

Outputs : Elevation(EL) & Azimuth(AZ) of Sun at EP

E16A 011. Elevation angle deg of Sun at EP using rv SEZ at UT time = -26.2490388254

E16B 011. Azimuth angle deg of sun at EP using rv SEZ at UT time = 255.0411079766

Elevation(EL) & Azimuth(AZ) angle of Sun at Earth Observation point EP, (method 2 - computed AZ values may be ambiguous & incorrect).

Inputs : Time input UT YY MM DD HH, Equator radius, EP lat & log, SS lat & log, Sun declination Delta

Outputs : Elevation(EL) & Azimuth(AZ) of Sun at EP

E16C 011. Elevation angle deg of Sun at EP using Sun declination diff log range EP to SC = 33.16712

E16D 011. Azimuth angle deg of sun at EP using sun declination diff log range EP to SC = -128.32056

Elevation(EL) & Azimuth(AZ) angle of Sun at Earth Observation point EP, (method 3 - computed AZ values may be ambiguous & incorrect).

Inputs : Time input UT YY MM DD HH, Equator radius, EP lat & log, SS lat & log, Sun hgt from EC, Sun range from EP

(Sun hgt from EC = earth orbit radius EC to SC km ; Sun range from EP = rv range vector EP to SC km frame SEZ)

Outputs : Elevation(EL) & Azimuth(AZ) of Sun at EP

E16E 011. Elevation angle deg of Sun at EP using Sun hgt diff log range EP to SC = 33.16375 ie deg = 33, min = 9, sec = 49.51

E16F 011. Azimuth angle deg of sun at EP using sun hgt diff log range EP to SC = 218.31935 ie deg = 218, min = 19, sec = 9.66

Azimuth(AZ) angle of Sun at Earth Observation point EP, (method 4 - computed AZ values is unambiguous & correct).

Inputs : Time input UT YY MM DD HH, EP lat & log, SS lat & log

Outputs : Azimuth(AZ) of Sun at EP

E16G 011. Azimuth angle deg of sun at EP using sun hgt diff log range EP to SC = 218.31935 ie deg = 218, min = 19, sec = 9.66

Due to such incorrect results, finally for Elevation (EL) Method 3 results and for Azimuth (AZ) Method 4 results are accepted.

Finally accepted Elevation angle deg of Sun from EP to SC = 33.1637517343 ie deg = 33, min = 9, sec = 49.51

Finally accepted Azimuth angle_deg of Sun from EP to SC = 218.3193510511 ie deg = 218, min = 19, sec = 9.66

Distance in km from Earth observation point(EP) to Sub Sun point(SS) and Earth Velocity meter per sec in orbit at time input UT.

17. Finding Distance in km from Earth observation point(EP) to Sub Sun point(SS) over Earth surface .

Inputs : EP lat & log, SS lat & log,

Outputs : Distance in km from EP to SS over Earth surface

E17A 011. Distance in km Earth observation point(EP) to Sub Sun point(SS) = 6326.74270

Finding Earth Velocity meter per sec in orbit in frame PQW

Inputs : semi-major axis SMA, GM_Sun, earth pos r EC to SC frame IJK, eccentricity of earth orbit e_Sun, sun eccentric anomaly E_Sun

Outputs : Earth Velocity magnitude and component Xw Yw in frame PQW in meter per sec

E17B 011. Velocity magnitude meter per sec using GM, SMA, r earth EC to SC frame IJK at UT time = 30286.5538639082

E17C 011. Velocity component meter per sec in orbit Xw using GM, e_Sun, SMA, E_Sun at UT time = -0.0000000000

E17D 011. Velocity component meter per sec in orbit Yw using GM, e_Sun, SMA, E_Sun at UT time = 30286.5538639082

Finding Earth Velocity Vector [vX, vY, vZ] in meter per sec in orbit; a Transform of [Xw, Yw] in frame PQW To [vX, vY, vZ] in frame XYZ

Inputs : velocity component (Xw, Yw), sun right ascension Alpha, Sun Argument of perigee W_Sun, inclination Epcylone

Outputs : earth velocity vector(vX, vY, vZ, vR) meter per sec in frame XYZ

E17E 011. vX earth Velocity vector meter per sec using Xw Yw frame PQW RA w i at UT time = 13409.1729584945

E17F 011. vY earth Velocity vector meter per sec using Xw Yw frame PQW RA w i at UT time = -27017.6074608137

E17G 011. vZ earth Velocity vector meter per sec using Xw Yw frame PQW RA w i at UT time = 2741.9541597144

E17H 011. vR earth Velocity magnitude meter per sec using Xw Yw frame PQW RA w i at UT time = 30286.5538639082

Earth State Vectors : Position [X, Y, Z] in km and Velocity [Vx, Vy, Vz] in meter per sec, at time input UT.

18. Finding Earth State Position Vector [X, Y, Z] in km at time input UT.

Inputs : position vector(r_I, r_J, r_K, r) in frame IJK values assigned to state position vector

Outputs : State Position Vector(X, Y, Z, R) in km, frame XYZ

E18A 011. State vector position X km at UT time = -119085952.6308768100

E18B 011. State vector position Y km at UT time = -64886279.6011658240

E18C 011. State vector position Z km at UT time = -56976844.6160544460

E18D 011. State vector position R km at UT time = 147098790.6710524900

19. Finding Earth State Velocity Vector [Vx, Vy, Vz] in meter per sec at time input UT.

Inputs : velocity vector(v_X, v_Y, v_Z, v_R) meter per sec in frame XYZ values assigned to state velocity vector

Outputs : state velocity vector(Vx, Vy, Vz, V) meter per sec, frame XYZ

E19A 011. State vector velocity Vx meter per sec at UT time = 13409.1729584945

E19B 011. State vector velocity Vy meter per sec at UT time = -27017.6074608137

E19C 011. State vector velocity Vz meter per sec at UT time = 2741.9541597144

019D 011. State vector velocity V meter per sec at UT time = 30286.5538639082

20. Earth Orbit Normal Vector [Wx, Wy, Wz] in km and angles Delta, i, RA at time input UT; Normal is line perpendicular to orbit plane.

Inputs : earth pos r EC to SC frame IJK, inclination Epcylone, sun right ascension Alpha

Outputs : earth orbit normal vector (Wx, Wy, Wz, W) in km

E20A 011. Earth orbit normal W km using r earth pos frame IJK inclination Alpha = 147098790.6710524900

E20B 011. Earth orbit normal Wx km using r earth pos frame IJK inclination Alpha = -56701506.3572354090

E20C 011. Earth orbit normal Wy km using r earth pos frame IJK inclination Alpha = -14444830.6291954550

E20D 011. Earth orbit normal Wz km using r earth pos frame IJK inclination Alpha = 134960513.7134575500

020E 011. Earth orbit normal Delta W deg using r earth pos frame IJK inclination Alpha = 66.5607205617

E20F 011. Earth orbit normal Inclination i deg using normal_Delta_W = 23.4392794383

E20G 011. Earth orbit normal Alpha W deg using r earth pos frame IJK, inclination, Alpha = 14.2922173002

E20H 011. Earth orbit normal Right ascension of ascending node using normal Alpha, W = 104.2922173002

Transform Earth State Vectors to Earth position Keplerian elements.

21. Finding Earth position Keplerian elements computed using State Vector, at time input UT.

Inputs : State vector year, days decimal of year, revolution, node, State Position Vector [X, Y, Z], State Velocity Vector [Vx, Vy, Vz]

Outputs : Keplerian elements : year, days decimal of year, revolution, node, inclination, right ascension, eccentricity, argument of perigee, mean anomaly, mean motion rev per day, mean angular velocity rev per day, mean motion rev_per_day from SMA considering oblateness

E21A 011. Keplerian elements year = 2013, days_decimal_of_year = 2.38329, revolution no = 1, node = 2 ie decending

E21B 011. inclination_deg = 23.4392794383

E21C 011. right ascension ascending node deg = 284.2922173002

E21D 011. eccentricity = 0.0167102190

E21E 011. argument of perigee_deg = 283.1557666033

E21F 011. mean anomaly deg = 0.0000000000

E21G 011. mean_motion rev per day = 0.0027377786

E21H 011. mean angular velocity rev_per_day = 0.0027377786

E21I 011. mean motion rev per day using SMA considering oblateness = 0.0027377786

Transform Earth position Keplerian elements to Earth State Vectors .

22. Finding Earth position State Vectors, computed using Keplerian elements at time input UT

(computed again to validate model equations, Keplerian elements to State Vectors & back)

Inputs : Keplerian elements : year, days decimal of year, revolution, node, inclination, right ascension, eccentricity, argument of perigee, mean anomaly, mean motion rev per day, mean angular velocity rev per day, mean motion rev_per_day from SMA considering oblateness

Outputs : State vector year, days decimal of year, revolution, node, State Position Vector [X, Y, Z], State Velocity Vector [Vx, Vy, Vz]

E22A 011. State vectors year = 2013, days_decimal_of_year = 2.38329, revolution no = 1, node = 2 ie decending

E22B 011. state vector position X km = -119085952.6308767500, state vector velocity Vx meter per sec = 13409.1729584945

E22C 011. state vector position Y km = -64886279.6011657040, state vector velocity Vy meter per sec = -27017.6074608137

E22D 011. state vector position Z km = -56976844.6160543860, state vector velocity Vz meter per sec = 2741.9541597144

E22E 011. state vector position R km = 147098790.6710523700, state vector velocity V meter per sec = 30286.5538639082

Note : Computation of all above parameters, grouped in 1 to 22, corresponds to time

(a) Universal time over Greenwich (UT/GMT) : Year = 2013, Month = 1, Day = 3, Hour = 9, Min = 11, Sec = 56.616

(b) Mean Solar time (MST) over Earth Observation point (EP) : Year = 2013, Month = 1, Day = 3, Hour = 14, Min = 21, Sec = 35.644

Move on to next Astronomical event in orbit Earth around Sun.

Next Section - 4.3 Position of earth at time when earth is at vernal equinox

OM-MSS Section - 4.3

Earth Positional Parameters on Celestial Sphere : Input Year Time when Earth is at Vernal equinox.

3. Find Position of Earth on Celestial Sphere at Input Universal Time, when Earth is at Vernal equinox .

Input UT Time, when Earth is at Vernal equinox : year = 2013, month = 3, day = 20, hour = 11, minute = 2, seconds = 9.15719
 Julian Day = 2456371.95983, year_day_decimal = 78.45983, day_hour_decimal = 11.03588
Observation Point on Earth (Bhopal, India) : Lat +ve or -ve 0 to 90 deg = 23.25993 ie deg = 23, min = 15, sec = 35.76
 Log 0 to 360 deg = 77.41261 ie deg = 77, min = 24, sec = 45.41
 Alt from earth surface in km = 0.49470

First Compute the **Sun Position** on Celestial Sphere, then Compute the **Earth Position** on Celestial Sphere.

(A) Computed Values for SUN POSITION on Celestial Sphere at Input Ut Time : (Sr. No 1 - 22)

01. Earth around Sun Mean motion rev per day (mm)	= 0.0027377786	02. Semi-major axis in km considering oblateness (SMA)	= 149598616.31172
03. Earth mean motion deg per day using SMA (mm)	= 0.9856003000	04. Sun mean movement deg per day (n sun)	= 0.9856003000
05. Eccentricity of earth orbit (e sun)	= 0.0167102190	06. Perihelion to input time diff in Julian days	= 76.0765340370
07. Mean anomaly in deg per day from n_sun (m sun)	= 74.9810547699	08. Sun Mean Longitude in deg (Lmean)	= 358.1404045779
09. Earth Mean anomaly in deg (ME)	= 74.9810547700	10. Sun Ecliptic Longitude in deg (Lsun)	= 0.0000000000
11. Obliquity of ecliptic in deg (Epcylone)	= 23.4375603478	12. Sun Right ascension in deg (Alpha)	= 0.0000000000
13. Sun Declination in deg (Delta)	= 0.0000000000	14. Sun Mean distance in km (As)	= 149598616.31172
15. Sun Radial distance from earth in km (Rs)	= 148989898.67840	16. Sun Nodal elongation in deg (U sun)	= 0.0000000000
17. Sun Mean anomaly in deg (M sun)	= 74.9810547697	18. Sun Eccentric anomaly in deg (E sun)	= 75.9096738744
19. Sun True anomaly in deg (T sun)	= 76.8402303407	20. Sun Argument of perigee in deg (W sun)	= 283.1597696594
21. Sun True anomaly in deg from U & W (V sun)	= 76.8402303407	22. Sun Distance in km (d sun)	= 148912015.96700

Sun Ecliptic latitude is always nearly zero (the value never exceeds 0.00033 deg)

These Values are applied as input for Computing Earth Position on Celestial Sphere around Sun at same input UT Time.

(B) Computed Values for EARTH POSITION on Celestial Sphere around Sun at same Input Ut Time : (Sr. No 1 - 22)

Input Time year = 2013, month = 3, day = 20, hour = 11, minute = 2, seconds = 9.15719, corresponding Julian Day = 2456371.9598282082
 Observation Point on Earth : Lat +ve or -ve 0 to 90 deg = 23.25993, Log 0 to 360 deg = 77.41261, Alt from earth surface in km = 0.49470
 Sun position on Celestial sphere at input time, computed above total 22 parameters.

Output Earth Position on Celestial sphere around Sun : **Computed below around 120 parameters, presented in 1-22 groups.**
 Number is large, because some parameters are computed using more than one model equation, that require different inputs.
 This helps in validation of results and understanding the different input considerations.

01. Finding GST Greenwich sidereal time and GHA Greenwich hour angle in 0 to 360 deg, at input UT time YY MM DD HH.

Note - for GST, the year 1900 JAN day_1 hr 1200 is ref for time difference in terms of julian_century,
 for GHA, the year 2000_JAN_day_1 hr_1200 is ref for time difference in terms of julian days.

Inputs : Time UT year = 2013, month = 3, day = 20, hour = 11, minute = 2, seconds = 9.15719

Outputs : GST & GHA in 0-360 deg over Greenwich.

E01A 011. GST Greenwich sidereal time in 0-360 deg, over Greenwich = 343.67866, hr = 22, min = 54, sec = 42.87769
 E01B 011. GHA Greenwich hour angle in 0 to 360 deg, over Greenwich = 343.68516, deg = 343, min = 41, sec = 6.56735

02. Finding Earth latitude & longitude pointing to Sun Ecliptic longitude(Lsun).

Inputs : earth inclination, sun true anomaly T_Sun, sun argument of perigee W_Sun, sun right ascension Alpha,
 earth equator radius, GST at input UT, log SS & EP, earth orbit radius EC to SC

Outputs : Earth lat & log pointing to Lsun.

E02A 011. Earth latitude +ve or -ve in 0 to 90 deg at UT time = 0.00 ie deg = 0, min = 0, sec = 0.00
 E02B 011. Earth longitude 0 to 360 deg = 16.32 ie deg = 16, min = 19, sec = 16.83

03. Finding LST over three longitudes, Greenwich log, Sun mean log (Lmean), and Sun ecliptic log (Lsun) .

Note - for LST, used sidereal time at Greenwich GST and desired geographic longitude

Inputs : At Time input UT - GST, Log of Greenwich, sun mean log Lmean, Sun ecliptic log Lsun.

Outputs : LST over Greenwich, Lmean, Lsun .

E03A 011. LST Local sidereal time in 0-360 deg, over Greenwich longitude = 343.67866, hr = 22, min = 54, sec = 42.87769

E03B 011. LST Local sidereal time in 0-360 deg, over Sun mean longitude (Lmean) = 341.81906, hr = 22, min = 47, sec = 16.57479
 E03C 011. LST Local sidereal time in 0-360 deg, over Sun epliptic longitude (Lsun) = 343.67866, hr = 22, min = 54, sec = 42.87769

04. Finding ST0 sidereal time over Greenwich longitude = 0.0, at time input Year JAN day 1 hr 00.

Note - this is sidereal time ST at UT year, month = 1, day = 1, hours decimal = 0.0 and geogrphic longitude = 0.0

Inputs : Time input UT Year, JAN day 1 hr 00, Log 0.0

Outputs : ST0 over Greenwich

E04 011. ST0 Sidereal time in 0-360 deg, over Greenwich at input UT year, MM 1, DD 1, HH 00 = 100.80678, hr = 6, min = 43, sec = 13.62710

05. Finding ST sidereal time over three longitudes of, Greenwich log, Sun mean log (Lmean), and Sun epliptic log (Lsun) .

Note - this is local sidereal time LST; (LST = GST at UT time + geogrphic longitude).

Inputs : At Time input UT - Log 0.0, Log Lmean, Log Lsun

Outputs : ST over Greenwich, Lmean, Lsun.

E05A 011. ST Sidereal time in 0-360 deg, over Greenwich at input UT time = 343.67866, hr = 22, min = 54, sec = 42.87769

E05B 011. ST Sidereal time in 0-360 deg, over Sun mean longitude (Lmean) at input UT time = 341.81906, hr = 22, min = 47, sec = 16.57479

005C 011. ST Sidereal time in 0-360 deg, over Sun longitude (Lsun) at input UT time = 343.67866, hr = 22, min = 54, sec = 42.87769

06. Finding H hour angle in 0 to 360 deg over longitudes of, Greenwich, Lmean, Lsun, Earth Sub Sun point SS, Earth Obseration point EP.

Note - used Sun Right ascension Alpha at input time; (hour angle HA = LST - Alpha).

Inputs : At Time input UT - Sun Right ascension Alpha and ST Sidereal time over longitudes 0.0, Lmean, Lsun, SS, EP

Outputs : Hour Angles over Greenwich, Lmean, Lsun, SS, EP

E06A 011. H hour angle 0-360 deg, over Greenwich, = 343.67866, deg = 343, min = 40, sec = 43.16538

E06B 011. H hour angle 0-360 deg, over Lmean, = 341.81906, deg = 341, min = 49, sec = 8.62186

E06C 011. H hour angle 0-360 deg, over Lsun, = 343.67866, deg = 343, min = 40, sec = 43.16538

E06D 011. H hour angle 0-360 deg, over SS, = 0.00000, deg = 0, min = 0, sec = 0.00000

E06E 011. H hour angle 0-360 deg, over EP, = 61.09127, deg = 61, min = 5, sec = 28.57938

07. Finding Delta E is Equation of Time in seconds, at time input UT.

Note - this value in seconds accounts for relative movement of sun in elliptical orbit w.r.t earth and effect of obliquity of the ecliptic;

its maximum value is 16 minutes (960 sec.); Delta E is computed using time in days from the perihelion, n_{sun_deg} and w_{sun} at input UT.

Inputs : Time input UT in JD, time perihelion in JD, Sun mean movement n_{sun} , Eccentricity of earth orbit E_{Sun}

Outputs : Delta E time_equation in seconds.

E07 011. Delta E Time Equation in seconds = 482.67883, hr = 0, min = 8, sec = 2.67883

08. Finding GST Greenwich sidereal time, and GHA Greenwich hour angle 0 to 360 deg at time when earth is at perihelion.

Inputs : Time in JD when earth at perihelion YY = 2013, MM = 1, DD = 3, hr = 9, min = 11, sec = 56.62

Outputs : GST & GHA in 0-360 deg over Greenwich when earth is at perihelion

E08A 011. GST sidereal time in 0-360 deg over Greenwich at time when earth is at perihelion = 241.14177, hr = 16, min = 4, sec = 34.02452

E08B 011. GHA hour angle in 0-360 deg over Greenwich at time when earth is at perihelion = 241.14717, hr = 16, min = 4, sec = 35.32064

09. Finding ST sidereal time and MST mean sidereal time, over Greenwich, using Earth mean motion rev per day .

Inputs : GST when earth at perihelion, earth rotation rate, ref. JD2000, time input UT in JD, time perihelion in JD.

Outputs : STP, angle perihelion to input JD, ST over Greenwich, MSTO & MST over Greenwich, solar time

E09A 011. STP sidereal time in 0-360 deg over Greenwich when earth at perihelion = 241.14177, hr = 16, min = 4, sec = 34.02452

E09B 011. Angle in 0-360 deg from earth at perihelion to input JD using earth rotational rate = 102.53422,

O09C 011. ST in 0-360 deg over Greenwich using STP and angle from perihelion at input JD = 343.67599, hr = 22, min = 54, sec = 42.23734

E09D 011. ST in 0-360 deg over Greenwich using STP and earth rotation at UT time = 343.67599, hr = 22, min = 54, sec = 42.23734

E09E 011. MSTO in deg, over Greenwich using JD century days, ref J2000 to I/P YY, M1, D1 hr 00 = 100.80714, hr = 6, min = 43, sec = 13.71450

E09F 011. MST in deg, over Greenwich using JD century in days from ref J2000 to UT time Y M D H = 343.67902, hr = 22, min = 54, sec = 42.96527

E09G 011. Solar time over Greenwich in JD (GMT or input UT - 12 hr) = YY 2013, MM 3, DD 19, hr 23, min 2, sec 9.157, ie JD 2456371.45983

10. Finding Earth orbit radius using true anomaly, Sub Sun point (SS) on earth surface and related parameters .

(a) Finding Earth orbit radius using true anomaly.

Inputs : semi-major axis SMA, eccentricity of earth orbit e_{sun} , sun true anomaly T_{Sun}

Outputs : earth orbital radius EC to SC (earth center to sun center)

E10A 011. earth orbital radius EC to SC km using true anomaly at UT time = 148990030.6242361400

(b) Finding Sub Sun point (SS) over earth surface (Latitude, Longitude, & Latitude radius) pointing to Sun Ecliptic Log (Lsun), Sun height from earth surface over SS, and LST over SS log at time input UT.

Note - for SS Latitude, used earth inclination, sun true anomaly T_sun and sun argument of perigee w_sun.
for SS Longitude, used Sun right ascension Alpha and sidereal time at Greenwich GST.

Inputs : earth inclination, sun true anomaly T_Sun, sun argument of perigee W_Sun, sun right ascension Alpha, earth equator radius, GST at input UT, log SS & EP, earth orbit radius EC to SC

Outputs : SS point Latitude, Longitude, Latitude radius, LST & LMT over SS .

E10B 011. SS point Latitude +ve or -ve in 0 to 90 deg at UT time = 0.00 ie deg = 0, min = 0, sec = 0.00

E10C 011. SS point Longitude 0 to 360 deg = 16.32 ie deg = 16, min = 19, sec = 16.83

E10D 011. SS point Latitude radius km at UT time = 6378.1440000000

E10E 011. Sun height km from earth surface over SS at UT time = 148983652.4802361400

E10F 011. LST local sidereal time in 0-360 deg over SS log at UT time, (LST = GST + log east) = 0.000 ie hr = 0, min = 0, sec = 0.08759
LST local sidereal time and LMT local mean time with date adjusted to calendar YY MM DD and UT hr mm sec.

E10G 011. LST local sidereal time at Sub Sun point (SS) YY = 2013, MM = 3, DD = 21, hr = 0, min = 0, sec = 0.09

E10H 011. LMT local Mean time at Sub Sun point (SS) YY = 2013, MM = 3, DD = 20, hr = 12, min = 7, sec = 26.28

(c) Finding LST and LMT over Earth point(EP) where Observer is, at time input UT.

Inputs : EP point Latitude, Longitude

Outputs : LST & LMT over EP .

E10I 011. LST local sidereal time in 0-360 deg at EP log at UT time, (LST = GST + log east) = 61.092 ie hr = 4, min = 4, sec = 21.99289
LST and LMT with date adjusted to calendar YY MM DD and UT hr mm sec.

E10J 011. LST local sidereal time at Earth point (EP) YY = 2013, MM = 3, DD = 21, hr = 4, min = 4, sec = 21.99

E10K 011. LMT local Mean time at Earth point (EP) YY = 2013, MM = 3, DD = 20, hr = 16, min = 11, sec = 48.18

Finding Earth to Sun Position Vectors coordinate in PQW, IJK, SEZ frames and the Vector Coordinate Transforms.

First defined coordinate systems, PQW, IJK, SEZ, then computed Position & Velocity vectors in these three coordinate systems.

(a) **Perifocal Coordinate System (POW)**, is Earth Centered Inertial coordinate frame defined in terms of Kepler Orbital Elements.

The system is fixed with time (inertial), pointing towards orbit periapsis;

the system's origin is Earth center (EC), and its fundamental plane is the orbit plane;

the P-vector axis directed from EC toward the periapsis of the elliptical orbit plane,

the Q-vector axis sweeps 90 deg from P axis in the direction of the orbit,

the W-vector axis directed from EC in a direction normal to orbit plane, forms a right-handed coordinate system.

(b) **Geocentric Coordinate System (IJK)**, is also an Earth Centered Inertial (ECI) frame, a Conventional Inertial System (CIS).

The system is fixed with time (inertial), pointing towards vernal equinox;

the system's origin is Earth center (EC), and its fundamental plane is the equator;

the I-vector is +X-axis directed towards the vernal equinox direction on J2000, Jan 1, hr 12.00 noon,

the J-vector is +Y-axis sweeps 90 deg to the east in the equatorial plane,

the K-vector is +Z-axis directed towards the North Pole.

(c) **Topocentric Horizon Coordinate System (SEZ)**, is Non-Inertial coordinate frame, known as Earth-Centered Earth-Fixed Coordinates (ECEF).

The system moves with earth, is not fixed with time (non-inertial), is for use by observers on the surface of earth;

the observer's surface forms the fundamental plane, is tangent to earth's surface

the S-vector is +ve horizontal -axis directed towards South,

the E-vector is +ve horizontal -axis directed towards East,

the Z-vector is +ve normal directed upwards on earth surface.

Note that axis Z not necessarily pass through earth center, so not used to define as radius vector.

11. Finding Earth center(EC) to Sun center(SC) Range Vector[r_p , r_q , r] from in PQW frame, perifocal coordinate system.

Inputs : Semi-major axis (SMA), Eccentricity of earth orbit (e_{sun}), Sun eccentric anomaly (E_{sun})

Outputs : Vector(r , r_p r_q) in PQW frame

E11A 011. r earth pos vector magnitude EC to SC km in PQW frame perifocal cord at UT time = 148990030.62424

E11B 011. rp earth pos vector component EC to SC km in PQW frame perifocal cord at UT time = 33920145.1258548950

E11C 011. rq earth pos vector component EC to SC km in PQW frame perifocal cord at UT time = 145077403.4095308500

Note - r earth pos vector magnitude EC to SC km in PQW frame is same as earth orbital radius computed before using true anomaly.

12. Transform_1 Earth position EC to SC Range Vector[r_p, r_q] in PQW frame To Range Vector[r_I, r_J, r_K] in IJK frame, inertial system cord.

Inputs : Vector(r_p, r_q) EC_to_SC km in frame PQW , Alpha rd, w_{sun} rd, earth_inclination rd ,

Outputs : Vector(r_I, r_J, r_K, r) EC_to_SC km in frame IJK

E12A 011. r_I earth pos vector component EC to SC km frame IJK at UT time = 148990030.6242361100

E12B 011. r_J earth pos vector component EC to SC km frame IJK at UT time = 0.0001317672

E12C 011. r_K earth pos vector component EC to SC km frame IJK at UT time = 0.0000285525

E12D 011. r earth pos vector magnitude EC to SC km frame IJK at UT time = 148990030.6242361100

Note - r earth pos vector magnitude EC to SC km in PQW frame is same as that computed above in PQW frame.

13. Transform_2 Earth point EP(lat, log, hgt) To EC to SC Range Vector[R_I, R_J, R_K, R] in IJK frame.

Inputs : earth equator radius_km, earth point EP(lat deg, log deg, hgt meter),

LST_local_sidereal_time_in_0_to_360_deg_at_EP_log_at_UT_time,

Outputs : Vector(R_I, R_J, R_K, R) Range EC to EP in IJK frame

E13A 011. R_I pos vector component EC to EP km frame IJK at UT time = 2831.4036789008

E13B 011. R_J pos vector component EC to EP km frame IJK at UT time = 5127.3099407798

E13C 011. R_K pos vector component EC to EP km frame IJK at UT time = 2517.6312937817

E13D 011. R pos vector magnitude EC to EP km frame IJK at UT time = 6375.3134317570

14. Transform_3 Earth position EC to SC Range Vectors [r_I, r_J, r_K] & [R_I, R_J, R_K] To EP to SC Range Vector[rv_I, rv_J, rv_K] in IJK frame.

Inputs : Vector(r_I, r_J, r_K) position EC_to_SC km in frame IJK , Vector(R_I, R_J, R_K) range EC to SC km in IJK frame,

Outputs : Vector(rv_I, rv_J, rv_K, rv) range EP to SC in IJK frame

E14A 011. rv_I range vector component EP to SC km frame IJK at UT time = 148987199.2205572100

E14B 011. rv_J range vector component EP to SC km frame IJK at UT time = -5127.3098090126

E14C 011. rv_K range vector component EP to SC km frame IJK at UT time = -2517.6312652292

E14D 011. rv range vector magnitude EP to SC km frame IJK at UT time = 148987199.3300558000

15. Transform_4 Earth point EP to SC Range Vector[rvI, rvJ, rvK] in IJK frame To EP to SC Range Vector[rvS, rvE, rvZ] in SEZ frame.

Inputs : lat_pos_neg_0_to_90_deg_at_EP_at_time_UT , LST_local_sidereal_time_in_0_to_360_deg_at_EP_log_at_UT_time

Vector(rvI, rvJ, rvK, rv) range EP to SC km in IJK frame,

Outputs : Vector(rvS, rvE, rvZ, rv) range EP to SC km in SEZ frame

E15A 011. rvS range vector component EP to SC km frame SEZ at UT time = 28442225.2836758790

E15B 011. rvE range vector component EP to SC km frame SEZ at UT time = -130424975.4876815500

E15C 011. rvZ range vector component EP to SC km frame SEZ at UT time = 66163064.8788587080

E15D 011. rv range vector magnitude EP to SC km frame SEZ at UT time = 148987199.3300557700

16. Finding Elevation(EL) and Azimuth(AZ) angle of Sun at Earth Observation point EP .

Note : Results **computed using 4 different formulations**, each require different inputs to give EL & AZ angles.

For all situations of Object and Observer positions, a combination of Latitude N/S & Longitude E/W :

Method 1 : for both EL & AZ angles, this does not provide correct results ;

Method 2 : for only EL angle, this provides consistent, unambiguous correct results.

but for AZ angles the results are ambiguous, need corrections by adding or subtracting values as 180 or 360 or sign change.

Method 3 : same as method 2, for EL angle, the results are correct, but for AZ angles the results are ambiguous, need corrections.

Method 4 : for finding Azimuth and Distance but not for finding Elevation angle;

for AZ angles, this provides correct unambiguous results that need no further corrections.

Therefore for Elevation (EL) angle Method 3 results are accepted and for Azimuth (AZ) angle Method 4 results are accepted .

Results verified from other sources; Ref URLs <http://www.ga.gov.au/geodesy/astro/smpos.jsp#intzone> .

NOAA Research <http://www.esrl.noaa.gov/gmd/grad/solcalc/> , and <http://aa.usno.navy.mil/data/docs/AltAz.php>

Xavier Jubier, Member IAU http://xjubier.free.fr/en/site_pages/astrometry/ephemerides.html

Rem: SS point lat deg = 0.00, log deg = 16.32 YY = 2013, MM = 3, DD = 20, hr = 12, min = 7, sec = 26.28

EP point lat deg = 23.26, log deg = 77.41 YY = 2013, MM = 3, DD = 20, hr = 16, min = 11, sec = 48.18

Elevation(EL) & Azimuth(AZ) angle of Sun at Earth Observation point EP, (**method 1** - computed values may be Ambiguous or Incorrect).

Inputs : Vector[rvS, rvE, rvZ] range EP to SC km in SEZ frame

Outputs : Elevation(EL) & Azimuth(AZ) of Sun at EP

E16A 011. Elevation angle deg of Sun at EP using rv SEZ at UT time = 26.3648484347

E16B 011. Azimuth angle deg of sun at EP using rv SEZ at UT time = 257.6979108329

Elevation(EL) & Azimuth(AZ) angle of Sun at Earth Observation point EP, (method 2 - computed AZ values may be ambiguous & incorrect).

Inputs : Time input UT YY MM DD HH, Equator radius, EP lat & log, SS lat & log, Sun declination Delta

Outputs : Elevation(EL) & Azimuth(AZ) of Sun at EP

E16C 011. Elevation angle deg of Sun at EP using Sun declination diff log range EP to SC = 26.36737

E16D 011. Azimuth angle deg of sun at EP using sun declination diff log range EP to SC = -167.69773

Elevation(EL) & Azimuth(AZ) angle of Sun at Earth Observation point EP, (method 3 - computed AZ values may be ambiguous & incorrect).

Inputs : Time input UT YY MM DD HH, Equator radius, EP lat & log, SS lat & log, Sun hgt from EC, Sun range from EP

(Sun hgt from EC = earth orbit radius EC to SC km ; Sun range from EP = rv range vector EP to SC km frame SEZ)

Outputs : Elevation(EL) & Azimuth(AZ) of Sun at EP

E16E 011. Elevation angle deg of Sun at EP using Sun hgt diff log range EP to SC = 26.36517 ie deg = 26, min = 21, sec = 54.63

E16F 011. Azimuth angle deg of sun at EP using sun hgt diff log range EP to SC = 257.69773 ie deg = 257, min = 41, sec = 51.83

Azimuth(AZ) angle of Sun at Earth Observation point EP, (method 4 - computed AZ values is unambiguous & correct).

Inputs : Time input UT YY MM DD HH, EP lat & log, SS lat & log

Outputs : Azimuth(AZ) of Sun at EP

E16G 011. Azimuth angle deg of sun at EP using sun hgt diff log range EP to SC = 257.69773 ie deg = 257, min = 41, sec = 51.83

Due to such incorrect results, finally for Elevation (EL) Method 3 results and for Azimuth (AZ) Method 4 results are accepted.

Finally accepted Elevation angle deg of Sun from EP to SC = 26.3651749316 ie deg = 26, min = 21, sec = 54.63

Finally accepted Azimuth angle_deg of Sun from EP to SC = 257.6977312752 ie deg = 257, min = 41, sec = 51.83

Distance in km from Earth observation point(EP) to Sub Sun point(SS) and Earth Velocity meter per sec in orbit at time input UT.

17. Finding Distance in km from Earth observation point(EP) to Sub Sun point(SS) over Earth surface .

Inputs : EP lat & log, SS lat & log,

Outputs : Distance in km from EP to SS over Earth surface

E17A 011. Distance in km Earth observation point(EP) to Sub Sun point(SS) = 7083.54272

Finding Earth Velocity meter per sec in orbit in frame PQW

Inputs : semi-major axis SMA, GM_Sun, earth pos r EC to SC frame IJK, eccentricity of earth orbit e_Sun, sun eccentric anomaly E_Sun

Outputs : Earth Velocity magnitude and component Xw Yw in frame PQW in meter per sec

E17B 011. Velocity magnitude meter per sec using GM, SMA, r earth EC to SC frame IJK at UT time = 29906.0325858305

E17C 011. Velocity component meter per sec in orbit Xw using GM, e_Sun, SMA, E_Sun at UT time = -29006.4938048237

E17D 011. Velocity component meter per sec in orbit Yw using GM, e_Sun, SMA, E_Sun at UT time = 7279.7048137597

Finding Earth Velocity Vector [vX, vY, vZ] in meter per sec in orbit; a Transform of [Xw, Yw] in frame PQW To [vX, vY, vZ] in frame XYZ

Inputs : velocity component (Xw, Yw), sun right ascension Alpha, Sun Argument of perigee W_Sun, inclination Epcylone

Outputs : earth velocity vector(vX, vY, vZ, vR) meter per sec in frame XYZ

E17E 011. vX earth Velocity vector meter per sec using Xw Yw frame PQW RA w i at UT time = 484.7048638742

E17F 011. vY earth Velocity vector meter per sec using Xw Yw frame PQW RA w i at UT time = 27434.6468079624

E17G 011. vZ earth Velocity vector meter per sec using Xw Yw frame PQW RA w i at UT time = 11894.3684465402

E17H 011. vR earth Velocity magnitude meter per sec using Xw Yw frame PQW RA w i at UT time = 29906.0325858305

Earth State Vectors : Position [X, Y, Z] in km and Velocity [Vx, Vy, Vz] in meter per sec, at time input UT.

18. Finding Earth State Position Vector [X, Y, Z] in km at time input UT.

Inputs : position vector(r_I, r_J, r_K, r) in frame IJK values assigned to state position vector

Outputs : State Position Vector(X, Y, Z, R) in km, frame XYZ

E18A 011. State vector position X km at UT time = 148990030.6242361100

E18B 011. State vector position Y km at UT time = 0.0001317672

E18C 011. State vector position Z km at UT time = 0.0000285525

E18D 011. State vector position R km at UT time = 148990030.6242361100

19. Finding Earth State Velocity Vector [Vx, Vy, Vz] in meter per sec at time input UT.

Inputs : velocity vector(v_X, v_Y, v_Z, v_R) meter per sec in frame XYZ values assigned to state velocity vector

Outputs : state velocity vector(Vx, Vy, Vz, V) meter per sec, frame XYZ

E19A 011. State vector velocity Vx meter per sec at UT time = 484.7048638742

E19B 011. State vector velocity Vy meter per sec at UT time = 27434.6468079624

E19C 011. State vector velocity Vz meter per sec at UT time = 11894.3684465402

019D 011. State vector velocity V meter per sec at UT time = 29906.0325858305

20. Earth Orbit Normal Vector [Wx, Wy, Wz] in km and angles Delta, i, RA at time input UT; Normal is line perpendicular to orbit plane.

Inputs : earth pos r EC to SC frame IJK, inclination Epcylone, sun right ascension Alpha

Outputs : earth orbit normal vector (Wx, Wy, Wz, W) in km

E20A 011. Earth orbit normal W km using r earth pos frame IJK inclination Alpha = 148990030.6242361100

E20B 011. Earth orbit normal Wx km using r earth pos frame IJK inclination Alpha = 0.0000262177

E20C 011. Earth orbit normal Wy km using r earth pos frame IJK inclination Alpha = -59264802.7950436320

E20D 011. Earth orbit normal Wz km using r earth pos frame IJK inclination Alpha = 136695692.5988357700

020E 011. Earth orbit normal Delta W deg using r earth pos frame IJK inclination Alpha = 66.5607205617

E20F 011. Earth orbit normal Inclination i deg using normal_Delta_W = 23.4392794383

E20G 011. Earth orbit normal Alpha W deg using r earth pos frame IJK, inclination, Alpha = -90.0000000000

E20H 011. Earth orbit normal Right ascension of ascending node using normal Alpha, W = 0.0000000000

Transform Earth State Vectors to Earth position Keplerian elements.

21. Finding Earth position Keplerian elements computed using State Vector, at time input UT.

Inputs : State vector year, days decimal of year, revolution, node, State Position Vector [X, Y, Z], State Velocity Vector [Vx, Vy, Vz]

Outputs : Keplerian elements : year, days decimal of year, revolution, node, inclination, right ascension, eccentricity, argument of perigee, mean anomaly, mean motion rev per day, mean angular velocity rev per day, mean motion rev_per_day from SMA considering oblateness

E21A 011. Keplerian elements year = 2013, days_decimal_of_year = 78.45983, revolution no = 1, node = 1 ie ascending

E21B 011. inclination_deg = 23.4392794383

E21C 011. right ascension ascending node deg = 0.0000000000

E21D 011. eccentricity = 0.0167102190

E21E 011. argument of perigee_deg = 283.1597696594

E21F 011. mean anomaly deg = 74.9810547697

E21G 011. mean_motion rev per day = 0.0027377786

E21H 011. mean angular velocity rev_per_day = 0.0027377786

E21I 011. mean motion rev per day using SMA considering oblateness = 0.0027377786

Transform Earth position Keplerian elements to Earth State Vectors .

22. Finding Earth position State Vectors, computed using Keplerian elements at time input UT

(computed again to validate model equations, Keplerian elements to State Vectors & back)

Inputs : Keplerian elements : year, days decimal of year, revolution, node, inclination, right ascension, eccentricity, argument of perigee, mean anomaly, mean motion rev per day, mean angular velocity rev per day, mean motion rev_per_day from SMA considering oblateness

Outputs : State vector year, days decimal of year, revolution, node, State Position Vector [X, Y, Z], State Velocity Vector [Vx, Vy, Vz]

E22A 011. State vectors year = 2013, days_decimal_of_year = 78.45983, revolution no = 1, node = 1 ie descending

E22B 011. state vector position X km = 148990030.6242359900, state vector velocity Vx meter per sec = 484.7048638741

E22C 011. state vector position Y km = 0.0001320988, state vector velocity Vy meter per sec = 27434.6468079624

E22D 011. state vector position Z km = 0.0000286959, state vector velocity Vz meter per sec = 11894.3684465402

E22E 011. state vector position R km = 148990030.6242359900, state vector velocity V meter per sec = 29906.0325858305

Note : Computation of all above parameters, grouped in 1 to 22, corresponds to time

(a) Universal time over Greenwich (UT/GMT) : Year = 2013, Month = 3, Day = 20, Hour = 11, Min = 2, Sec = 9.157

(b) Mean Solar time (MST) over Earth Observation point (EP) : Year = 2013, Month = 3, Day = 20, Hour = 16, Min = 11, Sec = 48.185

Move on to next Astronomical event in orbit Earth around Sun.

Next Section - 4.4 Position of earth at time when earth is at summer solstice

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Earth Positional Parameters on Celestial Sphere : Input Year Time when Earth is at Summer solstice.

4. Find Position of Earth on Celestial Sphere at Input Universal Time, when Earth is at Summer Solstice .

Input UT Time, when Earth is at Summer solstice : year = 2013, month = 6, day = 21, hour = 5, minute = 1, seconds = 19.19999
 Julian Day = 2456464.70925, year_day_decimal = 171.20925, day_hour_decimal = 5.02200

Observation Point on Earth (Bhopal, India) : Lat +ve or -ve 0 to 90 deg = 23.25993 ie deg = 23, min = 15, sec = 35.76
 Log 0 to 360 deg = 77.41261 ie deg = 77, min = 24, sec = 45.41
 Alt from earth surface in km = 0.49470

First Compute the **Sun Position** on Celestial Sphere, then Compute the **Earth Position** on Celestial Sphere.

(A) Computed Values for SUN POSITION on Celestial Sphere at Input Ut Time : (Sr. No 1 - 22)

01. Earth around Sun Mean motion rev per day (mm)	= 0.0027377786	02. Semi-major axis in km considering oblateness (SMA)	= 149598616.31172
03. Earth mean motion deg per day using SMA (mm)	= 0.9856003000	04. Sun mean movement deg per day (n sun)	= 0.9856003000
05. Eccentricity of earth orbit (e sun)	= 0.0167102190	06. Perihelion to input time diff in Julian days	= 168.8259558287
07. Mean anomaly in deg per day from n_sun (m sun)	= 166.3949127126	08. Sun Mean Longitude in deg (Lmean)	= 89.5586310183
09. Earth Mean anomaly in deg (ME)	= 166.3949127126	10. Sun Ecliptic Longitude in deg (Lsun)	= 89.9999483110
11. Obliquity of ecliptic in deg (Epcylone)	= 23.4375273104	12. Sun Right ascension in deg (Alpha)	= 89.9999436629
13. Sun Declination in deg (Delta)	= 23.4375273104	14. Sun Mean distance in km (As)	= 149598616.31172
15. Sun Radial distance from earth in km (Rs)	= 152030583.04072	16. Sun Nodal elongation in deg (U sun)	= 90.0000000000
17. Sun Mean anomaly in deg (M sun)	= 166.3949127122	18. Sun Eccentric anomaly in deg (E sun)	= 166.6165253213
19. Sun True anomaly in deg (T sun)	= 166.8363660940	20. Sun Argument of perigee in deg (W sun)	= 283.1636339060
21. Sun True anomaly in deg from U & W (V sun)	= 166.8363660940	22. Sun Distance in km (d sun)	= 152025947.60113

Sun Ecliptic latitude is always nearly zero (the value never exceeds 0.00033 deg)

These Values are applied as input for Computing Earth Position on Celestial Sphere around Sun at same input UT Time.

(B) Computed Values for EARTH POSITION on Celestial Sphere around Sun at same Input Ut Time : (Sr. No 1 - 22)

Input Time year = 2013, month = 6, day = 21, hour = 5, minute = 1, seconds = 19.19999, corresponding Julian Day = 2456464.7092499998
 Observation Point on Earth : Lat +ve or -ve 0 to 90 deg = 23.25993, Log 0 to 360 deg = 77.41261, Alt from earth surface in km = 0.49470
 Sun position on Celestial sphere at input time, computed above total 22 parameters.

Output Earth Position on Celestial sphere around Sun : **Computed below around 120 parameters, presented in 1-22 groups.**
 Number is large, because some parameters are computed using more than one model equation, that require different inputs.
 This helps in validation of results and understanding the different input considerations.

01. Finding GST Greenwich sidereal time and GHA Greenwich hour angle in 0 to 360 deg, at input UT time YY MM DD HH.

Note - for GST, the year 1900 JAN day_1 hr 1200 is ref for time difference in terms of julian_century,
 for GHA, the year 2000_JAN_day_1 hr_1200 is ref for time difference in terms of julian days.

Inputs : Time UT year = 2013, month = 6, day = 21, hour = 5, minute = 1, seconds = 19.19999

Outputs : GST & GHA in 0-360 deg over Greenwich.

E01A 011. GST Greenwich sidereal time in 0-360 deg, over Greenwich = 344.88872, hr = 22, min = 59, sec = 33.29393
 E01B 011. GHA Greenwich hour angle in 0 to 360 deg, over Greenwich = 344.89162, deg = 344, min = 53, sec = 29.83101

02. Finding Earth Latitude & Longitude pointing to Sun Ecliptic Longitude(Lsun).

Inputs : earth inclination, sun true anomaly T_Sun, sun argument of perigee W_Sun, sun right ascension Alpha,
 earth equator radius, GST at input UT, log SS & EP, earth orbit radius EC to SC

Outputs : Earth Lat & Log pointing to Lsun.

E02A 011. Earth Latitude +ve or -ve in 0 to 90 deg at UT time = 23.44 ie deg = 23, min = 26, sec = 21.41
 E02B 011. Earth Longitude 0 to 360 deg = 105.11 ie deg = 105, min = 6, sec = 40.39

03. Finding LST over three longitudes, Greenwich log, Sun mean log (Lmean), and Sun ecliptic log (Lsun) .

Note - for LST, used sidereal time at Greenwich GST and desired geographic longitude

Inputs : At Time input UT - GST, Log of Greenwich, sun mean log Lmean, Sun ecliptic log Lsun.

Outputs : LST over Greenwich, Lmean, Lsun .

E03A 011. LST Local sidereal time in 0-360 deg, over Greenwich longitude = 344.88872, hr = 22, min = 59, sec = 33.29393

E03B 011. LST Local sidereal time in 0-360 deg, over Sun mean longitude (Lmean) = 74.44736, hr = 4, min = 57, sec = 47.36538
 E03C 011. LST Local sidereal time in 0-360 deg, over Sun epliptic longitude (Lsun) = 74.88867, hr = 4, min = 59, sec = 33.28153

04. Finding ST0 sidereal time over Greenwich longitude = 0.0, at time input Year JAN day 1 hr 00.

Note - this is sidereal time ST at UT year, month = 1, day = 1, hours decimal = 0.0 and geographic longitude = 0.0

Inputs : Time input UT Year, JAN day 1 hr 00, Log 0.0

Outputs : ST0 over Greenwich

E04 011. ST0 Sidereal time in 0-360 deg, over Greenwich at input UT year, MM 1, DD 1, HH 00 = 100.80678, hr = 6, min = 43, sec = 13.62710

05. Finding ST sidereal time over three longitudes of, Greenwich log, Sun mean log (Lmean), and Sun epliptic log (Lsun) .

Note - this is local sidereal time LST; (LST = GST at UT time + geographic longitude).

Inputs : At Time input UT - Log 0.0, Log Lmean, Log Lsun

Outputs : ST over Greenwich, Lmean, Lsun.

E05A 011. ST Sidereal time in 0-360 deg, over Greenwich at input UT time = 344.88872, hr = 22, min = 59, sec = 33.29393

E05B 011. ST Sidereal time in 0-360 deg, over Sun mean longitude (Lmean) at input UT time = 74.44736, hr = 4, min = 57, sec = 47.36538

005C 011. ST Sidereal time in 0-360 deg, over Sun longitude (Lsun) at input UT time = 74.88867, hr = 4, min = 59, sec = 33.28153

06. Finding H hour angle in 0 to 360 deg over longitudes of, Greenwich, Lmean, Lsun, Earth Sub Sun point SS, Earth Obseration point EP.

Note - used Sun Right ascension Alpha at input time; (hour angle HA = LST - Alpha).

Inputs : At Time input UT - Sun Right ascension Alpha and ST Sidereal time over longitudes 0.0, Lmean, Lsun, SS, EP

Outputs : Hour Angles over Greenwich, Lmean, Lsun, SS, EP

E06A 011. H hour angle 0-360 deg, over Greenwich, = 254.88878, deg = 254, min = 53, sec = 19.61183

E06B 011. H hour angle 0-360 deg, over Lmean, = 344.44741, deg = 344, min = 26, sec = 50.68350

E06C 011. H hour angle 0-360 deg, over Lsun, = 344.88873, deg = 344, min = 53, sec = 19.42575

E06D 011. H hour angle 0-360 deg, over SS, = 360.00000, deg = 360, min = 0, sec = 0.00000

E06E 011. H hour angle 0-360 deg, over EP, = 332.30140, deg = 332, min = 18, sec = 5.02583

07. Finding Delta E is Equation of Time in seconds, at time input UT.

Note - this value in seconds accounts for relative movement of sun in elliptical orbit w.r.t earth and effect of obliquity of the ecliptic;

its maximum value is 16 minutes (960 sec.); Delta E is computed using time in days from the perihelion, n_sun_deg and w_sun at input UT.

Inputs : Time input UT in JD, time perihelion in JD, Sun mean movement n_sun, Eccentricity of earth orbit E_Sun

Outputs : Delta E time_equation in seconds.

E07 011. Delta E Time Equation in seconds = 99.08293, hr = 0, min = 1, sec = 39.08293

08. Finding GST Greenwich sidereal time, and GHA Greenwich hour angle 0 to 360 deg at time when earth is at perihelion.

Inputs : Time in JD when earth at perihelion YY = 2013, MM = 1, DD = 3, hr = 9, min = 11, sec = 56.62

Outputs : GST & GHA in 0-360 deg over Greenwich when earth is at perihelion

E08A 011. GST sidereal time in 0-360 deg over Greenwich at time when earth is at perihelion = 241.14177, hr = 16, min = 4, sec = 34.02452

E08B 011. GHA hour angle in 0-360 deg over Greenwich at time when earth is at perihelion = 241.14717, hr = 16, min = 4, sec = 35.32064

09. Finding ST sidereal time and MST mean sidereal time, over Greenwich, using Earth mean motion rev per day .

Inputs : GST when earth at perihelion, earth rotation rate, ref. JD2000, time input UT in JD, time perihelion in JD.

Outputs : STP, angle perihelion to input JD, ST over Greenwich, MSTO & MST over Greenwich, solar time

E09A 011. STP sidereal time in 0-360 deg over Greenwich when earth at perihelion = 241.14177, hr = 16, min = 4, sec = 34.02452

E09B 011. Angle in 0-360 deg from earth at perihelion to input JD using earth rotational rate = 103.74103,

O09C 011. ST in 0-360 deg over Greenwich using STP and angle from perihelion at input JD = 344.88280, hr = 22, min = 59, sec = 31.87289

E09D 011. ST in 0-360 deg over Greenwich using STP and earth rotation at UT time = 344.88280, hr = 22, min = 59, sec = 31.87289

E09E 011. MSTO in deg, over Greenwich using JD century days, ref J2000 to I/P YY, M1, D1 hr 00 = 100.80714, hr = 6, min = 43, sec = 13.71450

E09F 011. MST in deg, over Greenwich using JD century in days from ref J2000 to UT time Y M D H = 344.88909, hr = 22, min = 59, sec = 33.38172

E09G 011. Solar time over Greenwich in JD (GMT or input UT - 12 hr) = YY 2013, MM 6, DD 20, hr 17, min 1, sec 19.200, ie JD 2456464.20925

10. Finding Earth orbit radius using true anomaly, Sub Sun point (SS) on earth surface and related parameters .

(a) Finding Earth orbit radius using true anomaly.

Inputs : semi-major axis SMA, eccentricity of earth orbit e_sun, sun true anomaly T_Sun

Outputs : earth orbital radius EC to SC (earth center to sun center)

E10A 011. earth orbital radius EC to SC km using true anomaly at UT time = 152030553.3844403900

(b) Finding Sub Sun point (SS) over earth surface (Latitude, Longitude, & Latitude radius) pointing to Sun Ecliptic Log (Lsun), Sun height from earth surface over SS, and LST over SS log at time input UT.

Note - for SS Latitude, used earth inclination, sun true anomaly T_sun and sun argument of perigee w_sun.
for SS Longitude, used Sun right ascension Alpha and sidereal time at Greenwich GST.

Inputs : earth inclination, sun true anomaly T_Sun, sun argument of perigee W_Sun, sun right ascension Alpha, earth equator radius, GST at input UT, log SS & EP, earth orbit radius EC to SC

Outputs : SS point Latitude, Longitude, Latitude radius, LST & LMT over SS .

E10B 011. SS point Latitude +ve or -ve in 0 to 90 deg at UT time = 23.44 ie deg = 23, min = 26, sec = 21.41

E10C 011. SS point Longitude 0 to 360 deg = 105.11 ie deg = 105, min = 6, sec = 40.39

E10D 011. SS point Latitude radius km at UT time = 6374.7700931266

E10E 011. Sun height km from earth surface over SS at UT time = 152024178.6143472800

E10F 011. LST local sidereal time in 0-360 deg over SS log at UT time, (LST = GST + log east) = 90.000 ie hr = 6, min = 0, sec = 0.07427
LST local sidereal time and LMT local mean time with date adjusted to calendar YY MM DD and UT hr mm sec.

E10G 011. LST local sidereal time at Sub Sun point (SS) YY = 2013, MM = 6, DD = 22, hr = 6, min = 0, sec = 0.07

E10H 011. LMT local Mean time at Sub Sun point (SS) YY = 2013, MM = 6, DD = 21, hr = 12, min = 1, sec = 45.89

(c) Finding LST and LMT over Earth point(EP) where Observer is, at time input UT.

Inputs : EP point Latitude, Longitude

Outputs : LST & LMT over EP .

E10I 011. LST local sidereal time in 0-360 deg at EP log at UT time, (LST = GST + log east) = 62.302 ie hr = 4, min = 9, sec = 12.40934
LST and LMT with date adjusted to calendar YY MM DD and UT hr mm sec.

E10J 011. LST local sidereal time at Earth point (EP) YY = 2013, MM = 6, DD = 22, hr = 4, min = 9, sec = 12.41

E10K 011. LMT local Mean time at Earth point (EP) YY = 2013, MM = 6, DD = 21, hr = 10, min = 10, sec = 58.23

Finding Earth to Sun Position Vectors coordinate in PQW, IJK, SEZ frames and the Vector Coordinate Transforms.

First defined coordinate systems, PQW, IJK, SEZ, then computed Position & Velocity vectors in these three coordinate systems.

(a) **Perifocal Coordinate System (POW)**, is Earth Centered Inertial coordinate frame defined in terms of Kepler Orbital Elements.

The system is fixed with time (inertial), pointing towards orbit periapsis;

the system's origin is Earth center (EC), and its fundamental plane is the orbit plane;

the P-vector axis directed from EC toward the periapsis of the elliptical orbit plane,

the Q-vector axis sweeps 90 deg from P axis in the direction of the orbit,

the W-vector axis directed from EC in a direction normal to orbit plane, forms a right-handed coordinate system.

(b) **Geocentric Coordinate System (IJK)**, is also an Earth Centered Inertial (ECI) frame, a Conventional Inertial System (CIS).

The system is fixed with time (inertial), pointing towards vernal equinox;

the system's origin is Earth center (EC), and its fundamental plane is the equator;

the I-vector is +X-axis directed towards the vernal equinox direction on J2000, Jan 1, hr 12.00 noon,

the J-vector is +Y-axis sweeps 90 deg to the east in the equatorial plane,

the K-vector is +Z-axis directed towards the North Pole.

(c) **Topocentric Horizon Coordinate System (SEZ)**, is Non-Inertial coordinate frame, known as Earth-Centered Earth-Fixed Coordinates (ECEF).

The system moves with earth, is not fixed with time (non-inertial), is for use by observers on the surface of earth;

the observer's surface forms the fundamental plane, is tangent to earth's surface

the S-vector is +ve horizontal-axis directed towards South,

the E-vector is +ve horizontal-axis directed towards East,

the Z-vector is +ve normal directed upwards on earth surface.

Note that axis Z not necessarily pass through earth center, so not used to define as radius vector.

11. Finding Earth center(EC) to Sun center(SC) Range Vector[r_p , r_q , r] from in PQW frame, perifocal coordinate system.

Inputs : Semi-major axis (SMA), Eccentricity of earth orbit (e_{sun}), Sun eccentric anomaly (E_{sun})

Outputs : Vector(r , r_p r_q) in PQW frame

E11A 011. r earth pos vector magnitude EC to SC km in PQW frame perifocal cord at UT time = 152030553.38444

E11B 011. rp earth pos vector component EC to SC km in PQW frame perifocal cord at UT time = -148035744.2735802800

E11C 011. rq earth pos vector component EC to SC km in PQW frame perifocal cord at UT time = 34622356.6463585200

Note - r earth pos vector magnitude EC to SC km in PQW frame is same as earth orbital radius computed before using true anomaly.

12. Transform_1 Earth position EC to SC Range Vector[rp, rq] in PQW frame To Range Vector[rI, rJ, rK] in IJK frame, inertial system cord.

Inputs : Vector(rp, rq) EC_to_SC km in frame PQW , Alpha rd, w_sun rd, earth_inclination rd ,

Outputs : Vector(rI, rJ, rK, r) EC_to_SC km in frame IJK

E12A 011. rI earth pos vector component EC to SC km frame IJK at UT time = -139485317.9369729200

E12B 011. rJ earth pos vector component EC to SC km frame IJK at UT time = 137.1515086517

E12C 011. rK earth pos vector component EC to SC km frame IJK at UT time = 60474252.7228181590

E12D 011. r earth pos vector magnitude EC to SC km frame IJK at UT time = 152030553.3844447700

Note - r earth pos vector magnitude EC to SC km in PQW frame is same as that computed above in PQW frame.

13. Transform_2 Earth point EP(lat, log, hgt) To EC to SC Range Vector[RI, RJ, RK, R] in IJK frame.

Inputs : earth equator radius_km, earth point EP(lat deg, log deg, hgt meter),

LST_local_sidereal_time_in_0_to_360_deg_at_EP_log_at_UT_time,

Outputs : Vector(RI, RJ, RK, R) Range EC to EP in IJK frame

E13A 011. RI pos vector component EC to EP km frame IJK at UT time = 2722.4931452498

E13B 011. RJ pos vector component EC to EP km frame IJK at UT time = 5185.9603831673

E13C 011. RK pos vector component EC to EP km frame IJK at UT time = 2517.6312937817

E13D 011. R pos vector magnitude EC to EP km frame IJK at UT time = 6375.3134317570

14. Transform_3 Earth position EC to SC Range Vectors [rI rJ rK] & [RI RJ RK] To EP to SC Range Vector[rvI, rvJ, rvK] in IJK frame.

Inputs : Vector(rI rJ rK) position EC_to_SC km in frame IJK , Vector(RI RJ RK) range EC to SC km in IJK frame,

Outputs : Vector(rvI, rvJ, rvK, rv) range EP to SC in IJK frame

E14A 011. rvI range vector component EP to SC km frame IJK at UT time = -139488040.4301181700

E14B 011. rvJ range vector component EP to SC km frame IJK at UT time = -5048.8088745156

E14C 011. rvK range vector component EP to SC km frame IJK at UT time = 60471735.0915243770

E14D 011. rv range vector magnitude EP to SC km frame IJK at UT time = 152032049.8891739000

15. Transform_4 Earth point EP to SC Range Vector[rvI, rvJ, rvK] in IJK frame To EP to SC Range Vector[rvS, rvE, rvZ] in SEZ frame.

Inputs : lat_pos_neg_0_to_90_deg_at_EP_at_time_UT , LST_local_sidereal_time_in_0_to_360_deg_at_EP_log_at_UT_time

Vector(rvI, rvJ, rvK, rv) range EP to SC km in IJK frame,

Outputs : Vector(rvS, rvE, rvZ, rv) range EP to SC km in SEZ frame

E15A 011. rvS range vector component EP to SC km frame SEZ at UT time = -81162555.3582670990

E15B 011. rvE range vector component EP to SC km frame SEZ at UT time = 123501405.2118426000

E15C 011. rvZ range vector component EP to SC km frame SEZ at UT time = -35690148.6676749360

E15D 011. rv range vector magnitude EP to SC km frame SEZ at UT time = 152032049.8891738700

16. Finding Elevation(EL) and Azimuth(AZ) angle of Sun at Earth Observation point EP .

Note : Results computed using 4 different formulations, each require different inputs to give EL & AZ angles.

For all situations of Object and Observer positions, a combination of Latitude N/S & Longitude E/W :

Method 1 : for both EL & AZ angles, this does not provide correct results ;

Method 2 : for only EL angle, this provides consistent, unambiguous correct results.

but for AZ angles the results are ambiguous, need corrections by adding or subtracting values as 180 or 360 or sign change.

Method 3 : same as method 2, for EL angle, the results are correct, but for AZ angles the results are ambiguous, need corrections.

Method 4 : for finding Azimuth and Distance but not for finding Elevation angle;

for AZ angles, this provides correct unambiguous results that need no further corrections.

Therefore for Elevation (EL) angle Method 3 results are accepted and for Azimuth (AZ) angle Method 4 results are accepted .

Results verified from other sources; Ref URLs <http://www.ga.gov.au/geodesy/astro/smpos.jsp#intzone> .

NOAA Research <http://www.esrl.noaa.gov/gmd/grad/solcalc/> , and <http://aa.usno.navy.mil/data/docs/AltAz.php>

Xavier Jubier, Member IAU http://xjubier.free.fr/en/site_pages/astrometry/ephemerides.html

Rem: SS point lat deg = 23.44, log deg = 105.11 YY = 2013, MM = 6, DD = 21, hr = 12, min = 1, sec = 45.89

EP point lat deg = 23.26, log deg = 77.41 YY = 2013, MM = 6, DD = 21, hr = 10, min = 10, sec = 58.23

Elevation(EL) & Azimuth(AZ) angle of Sun at Earth Observation point EP, (method 1 - computed values may be Ambiguous or Incorrect).

Inputs : Vector[rvS, rvE, rvZ] range EP to SC km in SEZ frame

Outputs : Elevation(EL) & Azimuth(AZ) of Sun at EP

E16A 011. Elevation angle deg of Sun at EP using rv SEZ at UT time = -13.5771288889

E16B 011. Azimuth angle deg of sun at EP using rv SEZ at UT time = 56.6879146981

Elevation(EL) & Azimuth(AZ) angle of Sun at Earth Observation point EP, (method 2 - computed AZ values may be ambiguous & incorrect).

Inputs : Time input UT YY MM DD HH, Equator radius, EP lat & log, SS lat & log, Sun declination Delta

Outputs : Elevation(EL) & Azimuth(AZ) of Sun at EP

E16C 011. Elevation angle deg of Sun at EP using Sun declination diff log range EP to SC = 64.60870

E16D 011. Azimuth angle deg of sun at EP using sun declination diff log range EP to SC = 5.97289

Elevation(EL) & Azimuth(AZ) angle of Sun at Earth Observation point EP, (method 3 - computed AZ values may be ambiguous & incorrect).

Inputs : Time input UT YY MM DD HH, Equator radius, EP lat & log, SS lat & log, Sun hgt from EC, Sun range from EP

(Sun hgt from EC = earth orbit radius EC to SC km ; Sun range from EP = rv range vector EP to SC km frame SEZ)

Outputs : Elevation(EL) & Azimuth(AZ) of Sun at EP

E16E 011. Elevation angle deg of Sun at EP using Sun hgt diff log range EP to SC = 64.60783 ie deg = 64, min = 36, sec = 28.18

E16F 011. Azimuth angle deg of sun at EP using sun hgt diff log range EP to SC = 275.97696 ie deg = 275, min = 58, sec = 37.06

Azimuth(AZ) angle of Sun at Earth Observation point EP, (method 4 - computed AZ values is unambiguous & correct).

Inputs : Time input UT YY MM DD HH, EP lat & log, SS lat & log

Outputs : Azimuth(AZ) of Sun at EP

E16G 011. Azimuth angle deg of sun at EP using sun hgt diff log range EP to SC = 84.02304 ie deg = 84, min = 1, sec = 22.94

Due to such incorrect results, finally for Elevation (EL) Method 3 results and for Azimuth (AZ) Method 4 results are accepted.

Finally accepted Elevation angle deg of Sun from EP to SC = 64.6078265075 ie deg = 64, min = 36, sec = 28.18

Finally accepted Azimuth angle_deg of Sun from EP to SC = 84.0230377796 ie deg = 84, min = 1, sec = 22.94

Distance in km from Earth observation point(EP) to Sub Sun point(SS) and Earth Velocity meter per sec in orbit at time input UT.

17. Finding Distance in km from Earth observation point(EP) to Sub Sun point(SS) over Earth surface .

Inputs : EP lat & log, SS lat & log,

Outputs : Distance in km from EP to SS over Earth surface

E17A 011. Distance in km Earth observation point(EP) to Sub Sun point(SS) = 2826.52550

Finding Earth Velocity meter per sec in orbit in frame PQW

Inputs : semi-major axis SMA, GM_Sun, earth pos r EC to SC frame IJK, eccentricity of earth orbit e_Sun, sun eccentric anomaly E_Sun

Outputs : Earth Velocity magnitude and component Xw Yw in frame PQW in meter per sec

E17B 011. Velocity magnitude meter per sec using GM, SMA, r earth EC to SC frame IJK at UT time = 29304.2989199811

E17C 011. Velocity component meter per sec in orbit Xw using GM, e_Sun, SMA, E_Sun at UT time = -6783.8841216771

E17D 011. Velocity component meter per sec in orbit Yw using GM, e_Sun, SMA, E_Sun at UT time = -28508.2593543566

Finding Earth Velocity Vector [vX, vY, vZ] in meter per sec in orbit; a Transform of [Xw, Yw] in frame PQW To [vX, vY, vZ] in frame XYZ

Inputs : velocity component (Xw, Yw), sun right ascension Alpha, Sun Argument of perigee W_Sun, inclination Epcylone

Outputs : earth velocity vector(vX, vY, vZ, vR) meter per sec in frame XYZ

E17E 011. vX earth Velocity vector meter per sec using Xw Yw frame PQW RA w i at UT time = -104.0347630397

E17F 011. vY earth Velocity vector meter per sec using Xw Yw frame PQW RA w i at UT time = -29304.0795566878

E17G 011. vZ earth Velocity vector meter per sec using Xw Yw frame PQW RA w i at UT time = 45.0920724379

E17H 011. vR earth Velocity magnitude meter per sec using Xw Yw frame PQW RA w i at UT time = 29304.2989199811

Earth State Vectors : Position [X, Y, Z] in km and Velocity [Vx, Vy, Vz] in meter per sec, at time input UT.

18. Finding Earth State Position Vector [X, Y, Z] in km at time input UT.

Inputs : position vector(r_I, r_J, r_K, r) in frame IJK values assigned to state position vector

Outputs : State Position Vector(X, Y, Z, R) in km, frame XYZ

E18A 011. State vector position X km at UT time = -139485317.9369729200

E18B 011. State vector position Y km at UT time = 137.1515086517

E18C 011. State vector position Z km at UT time = 60474252.7228181590

E18D 011. State vector position R km at UT time = 152030553.3844447700

19. Finding Earth State Velocity Vector [Vx, Vy, Vz] in meter per sec at time input UT.

Inputs : velocity vector(v_X, v_Y, v_Z, v_R) meter per sec in frame XYZ values assigned to state velocity vector

Outputs : state velocity vector(Vx, Vy, Vz, V) meter per sec, frame XYZ

E19A 011. State vector velocity Vx meter per sec at UT time = -104.0347630397

E19B 011. State vector velocity Vy meter per sec at UT time = -29304.0795566878

E19C 011. State vector velocity Vz meter per sec at UT time = 45.0920724379

019D 011. State vector velocity V meter per sec at UT time = 29304.2989199811

20. Earth Orbit Normal Vector [Wx, Wy, Wz] in km and angles Delta, i, RA at time input UT; Normal is line perpendicular to orbit plane.

Inputs : earth pos r EC to SC frame IJK, inclination Epcylone, sun right ascension Alpha

Outputs : earth orbit normal vector (Wx, Wy, Wz, W) in km

E20A 011. Earth orbit normal W km using r earth pos frame IJK inclination Alpha = 152030553.3844447700

E20B 011. Earth orbit normal Wx km using r earth pos frame IJK inclination Alpha = 60474252.7227889370

E20C 011. Earth orbit normal Wy km using r earth pos frame IJK inclination Alpha = -59.4624231720

E20D 011. Earth orbit normal Wz km using r earth pos frame IJK inclination Alpha = 139485317.9370403600

020E 011. Earth orbit normal Delta W deg using r earth pos frame IJK inclination Alpha = 66.5607205617

E20F 011. Earth orbit normal Inclination i deg using normal_Delta_W = 23.4392794383

E20G 011. Earth orbit normal Alpha W deg using r earth pos frame IJK, inclination, Alpha = -0.0000563371

E20H 011. Earth orbit normal Right ascension of ascending node using normal Alpha, W = 89.9999436629

Transform Earth State Vectors to Earth position Keplerian elements.

21. Finding Earth position Keplerian elements computed using State Vector, at time input UT.

Inputs : State vector year, days decimal of year, revolution, node, State Position Vector [X, Y, Z], State Velocity Vector [Vx, Vy, Vz]

Outputs : Keplerian elements : year, days decimal of year, revolution, node, inclination, right ascension, eccentricity, argument of perigee, mean anomaly, mean motion rev per day, mean angular velocity rev per day, mean motion rev_per_day from SMA considering oblateness

E21A 011. Keplerian elements year = 2013, days_decimal_of_year = 171.20925, revolution no = 1, node = 1 ie ascending

E21B 011. inclination_deg = 23.4392794383

E21C 011. right ascension ascending node deg = 89.9999436629

E21D 011. eccentricity = 0.0167102190

E21E 011. argument of perigee_deg = 283.1636338813

E21F 011. mean anomaly deg = 166.3949127377

E21G 011. mean_motion rev per day = 0.0027377786

E21H 011. mean angular velocity rev_per_day = 0.0027377786

E21I 011. mean motion rev per day using SMA considering oblateness = 0.0027377786

Transform Earth position Keplerian elements to Earth State Vectors .

22. Finding Earth position State Vectors, computed using Keplerian elements at time input UT

(computed again to validate model equations, Keplerian elements to State Vectors & back)

Inputs : Keplerian elements : year, days decimal of year, revolution, node, inclination, right ascension, eccentricity, argument of perigee, mean anomaly, mean motion rev per day, mean angular velocity rev per day, mean motion rev_per_day from SMA considering oblateness

Outputs : State vector year, days decimal of year, revolution, node, State Position Vector [X, Y, Z], State Velocity Vector [Vx, Vy, Vz]

E22A 011. State vectors year = 2013, days_decimal_of_year = 171.20925, revolution no = 1, node = 1 ie descending

E22B 011. state vector position X km = -139485317.9369768200, state vector velocity Vx meter per sec = -104.0347628376

E22C 011. state vector position Y km = 137.1503847837, state vector velocity Vy meter per sec = -29304.0795566878

E22D 011. state vector position Z km = 60474252.7228198500, state vector velocity Vz meter per sec = 45.0920723503

E22E 011. state vector position R km = 152030553.3844490300, state vector velocity V meter per sec = 29304.2989199802

Note : Computation of all above parameters, grouped in 1 to 22, corresponds to time

(a) Universal time over Greenwich (UT/GMT) : Year = 2013, Month = 6, Day = 21, Hour = 5, Min = 1, Sec = 19.200

(b) Mean Solar time (MST) over Earth Observation point (EP) : Year = 2013, Month = 6, Day = 21, Hour = 10, Min = 10, Sec = 58.228

Move on to next Astronomical event in orbit Earth around Sun.

Next Section - 4.5 Position of earth at time when earth is at aphelion

Earth Positional Parameters on Celestial Sphere : Input Year Time when Earth is at Aphelion.

5. Finding Position of Earth on Celestial Sphere at Input Universal Time, when Earth is at Aphelion .

Input UT Time, when Earth is at Aphelion : year = 2013, month = 7, day = 5, hour = 0, minute = 18, seconds = 52.59269
 Julian Day = 2456478.51311, year_day_decimal = 185.01311, day_hour_decimal = 0.31461
 Observation Point on Earth (Bhopal, India) : Lat +ve or -ve 0 to 90 deg = 23.25993 ie deg = 23, min = 15, sec = 35.76
 Log 0 to 360 deg = 77.41261 ie deg = 77, min = 24, sec = 45.41
 Alt from earth surface in km = 0.49470

First Compute the Sun Position on Celestial Sphere, then Compute the Earth Position on Celestial Sphere.

(A) Computed Values for SUN POSITION on Celestial Sphere at Input Ut Time : (Sr. No 1 - 22)

01. Earth around Sun Mean motion rev per day (mm)	= 0.0027377786	02. Semi-major axis in km considering oblateness (SMA)	= 149598616.31172
03. Earth mean motion deg per day using SMA (mm)	= 0.9856003000	04. Sun mean movement deg per day (n sun)	= 0.9856003000
05. Eccentricity of earth orbit (e sun)	= 0.0167102190	06. Perihelion to input time diff in Julian days	= 182.6298145405
07. Mean anomaly in deg per day from n_sun (m sun)	= 180.0000000001	08. Sun Mean Longitude in deg (Lmean)	= 103.1643684676
09. Earth Mean anomaly in deg (ME)	= 180.0000000002	10. Sun Ecliptic Longitude in deg (Lsun)	= 103.1643684676
11. Obliquity of ecliptic in deg (Epcylone)	= 23.4375223935	12. Sun Right ascension in deg (Alpha)	= 104.3014954901
13. Sun Declination in deg (Delta)	= 22.7863704018	14. Sun Mean distance in km (As)	= 149598616.31172
15. Sun Radial distance from earth in km (Rs)	= 152098409.19029	16. Sun Nodal elongation in deg (U sun)	= 76.8356315324
17. Sun Mean anomaly in deg (M sun)	= 179.9999999997	18. Sun Eccentric anomaly in deg (E sun)	= 179.9999999997
19. Sun True anomaly in deg (T sun)	= 179.9999999997	20. Sun Argument of perigee in deg (W sun)	= 256.8356315327
21. Sun True anomaly in deg from U & W (V sun)	= 179.9999999997	22. Sun Distance in km (d sun)	= 152098441.95238

Sun Ecliptic latitude is always nearly zero (the value never exceeds 0.00033 deg)

These Values are applied as input for Computing Earth Position on Celestial Sphere around Sun at same input UT Time.

(B) Computed Values for EARTH POSITION on Celestial Sphere around Sun at same Input Ut Time : (Sr. No 1 - 22)

Input Time year = 2013, month = 7, day = 5, hour = 0, minute = 18, seconds = 52.59269, corresponding Julian Day = 2456478.5131087117
 Observation Point on Earth : Lat +ve or -ve 0 to 90 deg = 23.25993, Log 0 to 360 deg = 77.41261, Alt from earth surface in km = 0.49470
 Sun position on Celestial sphere at input time, computed above total 22 parameters.

Output Earth Position on Celestial sphere around Sun : **Computed below around 120 parameters, presented in 1-22 groups.**
 Number is large, because some parameters are computed using more than one model equation, that require different inputs.
 This helps in validation of results and understanding the different input considerations.

01. Finding GST Greenwich sidereal time and GHA Greenwich hour angle in 0 to 360 deg, at input UT time YY MM DD HH.

Note - for GST, the year 1900 JAN day_1 hr 1200 is ref for time difference in terms of julian_century,
 for GHA, the year 2000_JAN_day_1 hr_1200 is ref for time difference in terms of julian days.

Inputs : Time UT year = 2013, month = 7, day = 5, hour = 0, minute = 18, seconds = 52.59269

Outputs : GST & GHA in 0-360 deg over Greenwich.

E01A 011. GST Greenwich sidereal time in 0-360 deg, over Greenwich = 287.88360, hr = 19, min = 11, sec = 32.06349
 E01B 011. GHA Greenwich hour angle in 0 to 360 deg, over Greenwich = 287.88367, deg = 287, min = 53, sec = 1.21667

02. Finding Earth Latitude & Longitude pointing to Sun Ecliptic Longitude(Lsun).

Inputs : earth inclination, sun true anomaly T_Sun, sun argument of perigee W_Sun, sun right ascension Alpha,
 earth equator radius, GST at input UT, log SS & EP, earth orbit radius EC to SC

Outputs : Earth Lat & Log pointing to Lsun.

E02A 011. Earth Latitude +ve or -ve in 0 to 90 deg at UT time = 22.79 ie deg = 22, min = 47, sec = 17.06
 E02B 011. Earth Longitude 0 to 360 deg = 176.42 ie deg = 176, min = 25, sec = 4.43

03. Finding LST over three longitudes, Greenwich log, Sun mean log (Lmean), and Sun ecliptic log (Lsun) .

Note - for LST, used sidereal time at Greenwich GST and desired geographic longitude

Inputs : At Time input UT - GST, Log of Greenwich, sun mean log Lmean, Sun ecliptic log Lsun.

Outputs : LST over Greenwich, Lmean, Lsun .

E03A 011. LST Local sidereal time in 0-360 deg, over Greenwich longitude = 287.88360, hr = 19, min = 11, sec = 32.06349

E03B 011. LST Local sidereal time in 0-360 deg, over Sun mean longitude (Lmean) = 31.04797, hr = 2, min = 4, sec = 11.51192
 E03C 011. LST Local sidereal time in 0-360 deg, over Sun epliptic longitude (Lsun) = 31.04797, hr = 2, min = 4, sec = 11.51192

04. Finding ST0 sidereal time over Greenwich longitude = 0.0, at time input Year JAN day 1 hr 00.

Note - this is sidereal time ST at UT year, month = 1, day = 1, hours decimal = 0.0 and geogrphic longitude = 0.0

Inputs : Time input UT Year, JAN day 1 hr 00, Log 0.0

Outputs : ST0 over Greenwich

E04 011. ST0 Sidereal time in 0-360 deg, over Greenwich at input UT year, MM 1, DD 1, HH 00 = 100.80678, hr = 6, min = 43, sec = 13.62710

05. Finding ST sidereal time over three longitudes of, Greenwich log, Sun mean log (Lmean), and Sun epliptic log (Lsun) .

Note - this is local sidereal time LST; (LST = GST at UT time + geogrphic longitude).

Inputs : At Time input UT - Log 0.0, Log Lmean, Log Lsun

Outputs : ST over Greenwich, Lmean, Lsun.

E05A 011. ST Sidereal time in 0-360 deg, over Greenwich at input UT time = 287.88360, hr = 19, min = 11, sec = 32.06349

E05B 011. ST Sidereal time in 0-360 deg, over Sun mean longitude (Lmean) at input UT time = 31.04797, hr = 2, min = 4, sec = 11.51192

005C 011. ST Sidereal time in 0-360 deg, over Sun longitude (Lsun) at input UT time = 31.04797, hr = 2, min = 4, sec = 11.51192

06. Finding H hour angle in 0 to 360 deg over longitudes of, Greenwich, Lmean, Lsun, Earth Sub Sun point SS, Earth Obseration point EP.

Note - used Sun Right ascension Alpha at input time; (hour angle HA = LST - Alpha).

Inputs : At Time input UT - Sun Right ascension Alpha and ST Sidereal time over longitudes 0.0, Lmean, Lsun, SS, EP

Outputs : Hour Angles over Greenwich, Lmean, Lsun, SS, EP

E06A 011. H hour angle 0-360 deg, over Greenwich, = 183.58210, deg = 183, min = 34, sec = 55.56862

E06B 011. H hour angle 0-360 deg, over Lmean, = 286.74647, deg = 286, min = 44, sec = 47.29510

E06C 011. H hour angle 0-360 deg, over Lsun, = 286.74647, deg = 286, min = 44, sec = 47.29510

E06D 011. H hour angle 0-360 deg, over SS, = 0.00000, deg = 0, min = 0, sec = 0.00000

E06E 011. H hour angle 0-360 deg, over EP, = 260.99472, deg = 260, min = 59, sec = 40.98262

07. Finding Delta E is Equation of Time in seconds, at time input UT.

Note - this value in seconds accounts for relative movement of sun in elliptical orbit w.r.t earth and effect of obliquity of the ecliptic;

its maximum value is 16 minutes (960 sec.); Delta E is computed using time in days from the perihelion, n_sun_deg and w_sun at input UT.

Inputs : Time input UT in JD, time perihelion in JD, Sun mean movement n_sun, Eccentricity of earth orbit E_Sun

Outputs : Delta E time_equation in seconds.

E07 011. Delta E Time Equation in seconds = -262.56430, hr = 0, min = 4, sec = 22.56430

08. Finding GST Greenwich sidereal time, and GHA Greenwich hour angle 0 to 360 deg at time when earth is at perihelion.

Inputs : Time in JD when earth at perihelion YY = 2013, MM = 1, DD = 3, hr = 9, min = 11, sec = 56.62

Outputs : GST & GHA in 0-360 deg over Greenwich when earth is at perihelion

E08A 011. GST sidereal time in 0-360 deg over Greenwich at time when earth is at perihelion = 241.14177, hr = 16, min = 4, sec = 34.02452

E08B 011. GHA hour angle in 0-360 deg over Greenwich at time when earth is at perihelion = 241.14717, hr = 16, min = 4, sec = 35.32064

09. Finding ST sidereal time and MST mean sidereal time, over Greenwich, using Earth mean motion rev per day .

Inputs : GST when earth at perihelion, earth rotation rate, ref. JD2000, time input UT in JD, time perihelion in JD.

Outputs : STP, angle perihelion to input JD, ST over Greenwich, MSTO & MST over Greenwich, solar time

E09A 011. STP sidereal time in 0-360 deg over Greenwich when earth at perihelion = 241.14177, hr = 16, min = 4, sec = 34.02452

E09B 011. Angle in 0-360 deg from earth at perihelion to input JD using earth rotational rate = 46.73542,

O09C 011. ST in 0-360 deg over Greenwich using STP and angle from perihelion at input JD = 287.87719, hr = 19, min = 11, sec = 30.52626

E09D 011. ST in 0-360 deg over Greenwich using STP and earth rotation at UT time = 287.87719, hr = 19, min = 11, sec = 30.52626

E09E 011. MSTO in deg, over Greenwich using JD century days, ref J2000 to I/P YY, M1, D1 hr 00 = 100.80714, hr = 6, min = 43, sec = 13.71450

E09F 011. MST in deg, over Greenwich using JD century in days from ref J2000 to UT time Y M D H = 287.88396, hr = 19, min = 11, sec = 32.15131

E09G 011. Solar time over Greenwich in JD (GMT or input UT - 12 hr) = YY 2013, MM 7, DD 4, hr 12, min 18, sec 52.593, ie JD 2456478.01311

10. Finding Earth orbit radius using true anomaly, Sub Sun point (SS) on earth surface and related parameters .

(a) Finding Earth orbit radius using true anomaly.

Inputs : semi-major axis SMA, eccentricity of earth orbit e_sun, sun true anomaly T_Sun

Outputs : earth orbital radius EC to SC (earth center to sun center)

E10A 011. earth orbital radius EC to SC km using true anomaly at UT time = 152098441.9523840800

(b) Finding Sub Sun point (SS) over earth surface (Latitude, Longitude, & Latitude radius) pointing to Sun Ecliptic Log (Lsun), Sun height from earth surface over SS, and LST over SS log at time input UT.

Note - for SS Latitude, used earth inclination, sun true anomaly T_sun and sun argument of perigee w_sun.
for SS Longitude, used Sun right ascension Alpha and sidereal time at Greenwich GST.

Inputs : earth inclination, sun true anomaly T_Sun, sun argument of perigee W_Sun, sun right ascension Alpha, earth equator radius, GST at input UT, log SS & EP, earth orbit radius EC to SC

Outputs : SS point Latitude, Longitude, Latitude radius, LST & LMT over SS .

E10B 011. SS point Latitude +ve or -ve in 0 to 90 deg at UT time = 22.79 ie deg = 22, min = 47, sec = 17.06

E10C 011. SS point Longitude 0 to 360 deg = 176.42 ie deg = 176, min = 25, sec = 4.43

E10D 011. SS point Latitude radius km at UT time = 6374.9453113297

E10E 011. Sun height km from earth surface over SS at UT time = 152092067.0070727500

E10F 011. LST local sidereal time in 0-360 deg over SS log at UT time, (LST = GST + log east) = 104.302 ie hr = 6, min = 57, sec = 12.44676
LST local sidereal time and LMT local mean time with date adjusted to calendar YY MM DD and UT hr mm sec.

E10G 011. LST local sidereal time at Sub Sun point (SS) YY = 2013, MM = 7, DD = 6, hr = 6, min = 57, sec = 12.45

E10H 011. LMT local Mean time at Sub Sun point (SS) YY = 2013, MM = 7, DD = 5, hr = 12, min = 4, sec = 32.89

(c) Finding LST and LMT over Earth point(EP) where Observer is, at time input UT.

Inputs : EP point Latitude, Longitude

Outputs : LST & LMT over EP .

E10I 011. LST local sidereal time in 0-360 deg at EP log at UT time, (LST = GST + log east) = 5.297 ie hr = 0, min = 21, sec = 11.17894
LST and LMT with date adjusted to calendar YY MM DD and UT hr mm sec.

E10J 011. LST local sidereal time at Earth point (EP) YY = 2013, MM = 7, DD = 6, hr = 0, min = 21, sec = 11.18

E10K 011. LMT local Mean time at Earth point (EP) YY = 2013, MM = 7, DD = 5, hr = 5, min = 28, sec = 31.62

Finding Earth to Sun Position Vectors coordinate in PQW, IJK, SEZ frames and the Vector Coordinate Transforms.

First defined coordinate systems, PQW, IJK, SEZ, then computed Position & Velocity vectors in these three coordinate systems

(a) **Perifocal Coordinate System (POW)**, is Earth Centered Inertial coordinate frame defined in terms of Kepler Orbital Elements.

The system is fixed with time (inertial), pointing towards orbit periapsis;

the system's origin is Earth center (EC), and its fundamental plane is the orbit plane;

the P-vector axis directed from EC toward the periapsis of the elliptical orbit plane,

the Q-vector axis sweeps 90 deg from P axis in the direction of the orbit,

the W-vector axis directed from EC in a direction normal to orbit plane, forms a right-handed coordinate system.

(b) **Geocentric Coordinate System (IJK)**, is also an Earth Centered Inertial (ECI) frame, a Conventional Inertial System (CIS).

The system is fixed with time (inertial), pointing towards vernal equinox;

the system's origin is Earth center (EC), and its fundamental plane is the equator;

the I-vector is +X-axis directed towards the vernal equinox direction on J2000, Jan 1, hr 12.00 noon,

the J-vector is +Y-axis sweeps 90 deg to the east in the equatorial plane,

the K-vector is +Z-axis directed towards the North Pole.

(c) **Topocentric Horizon Coordinate System (SEZ)**, is Non-Inertial coordinate frame, known as Earth-Centered Earth-Fixed Coordinates (ECEF).

The system moves with earth, is not fixed with time (non-inertial), is for use by observers on the surface of earth;

the observer's surface forms the fundamental plane, is tangent to earth's surface

the S-vector is +ve horizontal-axis directed towards South,

the E-vector is +ve horizontal-axis directed towards East,

the Z-vector is +ve normal directed upwards on earth surface.

Note that axis Z not necessarily pass through earth center, so not used to define as radius vector.

11. Finding Earth center(EC) to Sun center(SC) Range Vector[r_p , r_q , r] from in PQW frame, perifocal coordinate system.

Inputs : Semi-major axis (SMA), Eccentricity of earth orbit (e_{sun}), Sun eccentric anomaly (E_{sun})

Outputs : Vector(r , r_p r_q) in PQW frame

E11A 011. r earth pos vector magnitude EC to SC km in PQW frame perifocal cord at UT time = 152098441.95238

E11B 011. rp earth pos vector component EC to SC km in PQW frame perifocal cord at UT time = -152098441.9523840500

E11C 011. rq earth pos vector component EC to SC km in PQW frame perifocal cord at UT time = 0.0007434219

Note - r earth pos vector magnitude EC to SC km in PQW frame is same as earth orbital radius computed before using true anomaly.

12. Transform_1 Earth position EC to SC Range Vector[r_p, r_q] in PQW frame To Range Vector[r_I, r_J, r_K] in IJK frame, inertial system cord.

Inputs : Vector(r_p, r_q) EC_to_SC km in frame PQW , Alpha rd, w_{sun} rd, earth_inclination rd ,

Outputs : Vector(r_I, r_J, r_K, r) EC_to_SC km in frame IJK

E12A 011. r_I earth pos vector component EC to SC km frame IJK at UT time = -140226215.3950415300

E12B 011. r_J earth pos vector component EC to SC km frame IJK at UT time = 446.2552520869

E12C 011. r_K earth pos vector component EC to SC km frame IJK at UT time = 58911327.9440127610

E12D 011. r earth pos vector magnitude EC to SC km frame IJK at UT time = 152098441.9523840500

Note - r earth pos vector magnitude EC to SC km in PQW frame is same as that computed above in PQW frame.

13. Transform_2 Earth point EP(lat, log, hgt) To EC to SC Range Vector[R_I, R_J, R_K, R] in IJK frame.

Inputs : earth equator radius_km, earth point EP(lat deg, log deg, hgt meter),

LST_local_sidereal_time_in_0_to_360_deg_at_EP_log_at_UT_time,

Outputs : Vector(R_I, R_J, R_K, R) Range EC to EP in IJK frame

E13A 011. R_I pos vector component EC to EP km frame IJK at UT time = 5832.1367811816

E13B 011. R_J pos vector component EC to EP km frame IJK at UT time = 540.6797455991

E13C 011. R_K pos vector component EC to EP km frame IJK at UT time = 2517.6312937817

E13D 011. R pos vector magnitude EC to EP km frame IJK at UT time = 6375.3134317570

14. Transform_3 Earth position EC to SC Range Vectors [r_I, r_J, r_K] & [R_I, R_J, R_K] To EP to SC Range Vector[rv_I, rv_J, rv_K] in IJK frame.

Inputs : Vector(r_I, r_J, r_K) position EC_to_SC km in frame IJK , Vector(R_I, R_J, R_K) range EC to SC km in IJK frame,

Outputs : Vector(rv_I, rv_J, rv_K, rv) range EP to SC in IJK frame

E14A 011. rv_I range vector component EP to SC km frame IJK at UT time = -140232047.5318227100

E14B 011. rv_J range vector component EP to SC km frame IJK at UT time = -94.4244935123

E14C 011. rv_K range vector component EP to SC km frame IJK at UT time = 58908810.3127189800

E14D 011. rv range vector magnitude EP to SC km frame IJK at UT time = 152102843.7848425200

15. Transform_4 Earth point EP to SC Range Vector[rvI, rvJ, rvK] in IJK frame To EP to SC Range Vector[rvS, rvE, rvZ] in SEZ frame.

Inputs : lat_pos_neg_0_to_90_deg_at_EP_at_time_UT , LST_local_sidereal_time_in_0_to_360_deg_at_EP_log_at_UT_time

Vector(rvI, rvJ, rvK, rv) range EP to SC km in IJK frame,

Outputs : Vector(rvS, rvE, rvZ, rv) range EP to SC km in SEZ frame

E15A 011. rvS range vector component EP to SC km frame SEZ at UT time = -109262491.5112520900

E15B 011. rvE range vector component EP to SC km frame SEZ at UT time = 12944885.2533702720

E15C 011. rvZ range vector component EP to SC km frame SEZ at UT time = -105021012.0974219600

E15D 011. rv range vector magnitude EP to SC km frame SEZ at UT time = 152102843.7848425200

16. Finding Elevation(EL) and Azimuth(AZ) angle of Sun at Earth Observation point EP .

Note : Results **computed using 4 different formulations**, each require different inputs to give EL & AZ angles.

For all situations of Object and Observer positions, a combination of Latitude N/S & Longitude E/W :

Method 1 : for both EL & AZ angles, this does not provide correct results ;

Method 2 : for only EL angle, this provides consistent, unambiguous correct results.

but for AZ angles the results are ambiguous, need corrections by adding or subtracting values as 180 or 360 or sign change.

Method 3 : same as method 2, for EL angle, the results are correct, but for AZ angles the results are ambiguous, need corrections.

Method 4 : for finding Azimuth and Distance but not for finding Elevation angle;

for AZ angles, this provides correct unambiguous results that need no further corrections.

Therefore for Elevation (EL) angle Method 3 results are accepted and for Azimuth (AZ) angle Method 4 results are accepted .

Results verified from other sources; Ref URLs <http://www.ga.gov.au/geodesy/astro/smpos.jsp#intzone> .

NOAA Research <http://www.esrl.noaa.gov/gmd/grad/solcalc/> , and <http://aa.usno.navy.mil/data/docs/AltAz.php>

Xavier Jubier, Member IAU http://xjubier.free.fr/en/site_pages/astronomy/ephemerides.html

Rem: SS point lat deg = 22.79, log deg = 176.42 YY = 2013, MM = 7, DD = 5, hr = 12, min = 4, sec = 32.89

EP point lat deg = 23.26, log deg = 77.41 YY = 2013, MM = 7, DD = 5, hr = 5, min = 28, sec = 31.62

Elevation(EL) & Azimuth(AZ) angle of Sun at Earth Observation point EP, (**method 1** - computed values may be Ambiguous or Incorrect).

Inputs : Vector[rvS, rvE, rvZ] range EP to SC km in SEZ frame

Outputs : Elevation(EL) & Azimuth(AZ) of Sun at EP

E16A 011. Elevation angle deg of Sun at EP using rv SEZ at UT time = -43.6665758927

E16B 011. Azimuth angle deg of sun at EP using rv SEZ at UT time = 6.7566282101

Elevation(EL) & Azimuth(AZ) angle of Sun at Earth Observation point EP, (method 2 - computed AZ values may be ambiguous & incorrect).

Inputs : Time input UT YY MM DD HH, Equator radius, EP lat & log, SS lat & log, Sun declination Delta

Outputs : Elevation(EL) & Azimuth(AZ) of Sun at EP

E16C 011. Elevation angle deg of Sun at EP using Sun declination diff log range EP to SC = 1.16686

E16D 011. Azimuth angle deg of sun at EP using sun declination diff log range EP to SC = 24.38662

Elevation(EL) & Azimuth(AZ) angle of Sun at Earth Observation point EP, (method 3 - computed AZ values may be ambiguous & incorrect).

Inputs : Time input UT YY MM DD HH, Equator radius, EP lat & log, SS lat & log, Sun hgt from EC, Sun range from EP

(Sun hgt from EC = earth orbit radius EC to SC km ; Sun range from EP = rv range vector EP to SC km frame SEZ)

Outputs : Elevation(EL) & Azimuth(AZ) of Sun at EP

E16E 011. Elevation angle deg of Sun at EP using Sun hgt diff log range EP to SC = 1.16517 ie deg = 1, min = 9, sec = 54.60

E16F 011. Azimuth angle deg of sun at EP using sun hgt diff log range EP to SC = 245.61183 ie deg = 245, min = 36, sec = 42.59

Azimuth(AZ) angle of Sun at Earth Observation point EP, (method 4 - computed AZ values is unambiguous & correct).

Inputs : Time input UT YY MM DD HH, EP lat & log, SS lat & log

Outputs : Azimuth(AZ) of Sun at EP

E16G 011. Azimuth angle deg of sun at EP using sun hgt diff log range EP to SC = 65.61183 ie deg = 65, min = 36, sec = 42.59

Due to such incorrect results, finally for Elevation (EL) Method 3 results and for Azimuth (AZ) Method 4 results are accepted.

Finally accepted Elevation angle deg of Sun from EP to SC = 1.1651677095 ie deg = 1, min = 9, sec = 54.60

Finally accepted Azimuth angle_deg of Sun from EP to SC = 65.6118300528 ie deg = 65, min = 36, sec = 42.59

Distance in km from Earth observation point(EP) to Sub Sun point(SS) and Earth Velocity meter per sec in orbit at time input UT.

17. Finding Distance in km from Earth observation point(EP) to Sub Sun point(SS) over Earth surface .

Inputs : EP lat & log, SS lat & log,

Outputs : Distance in km from EP to SS over Earth surface

E17A 011. Distance in km Earth observation point(EP) to Sub Sun point(SS) = 9888.76838

Finding Earth Velocity meter per sec in orbit in frame PQW

Inputs : semi-major axis SMA, GM_Sun, earth pos r EC to SC frame IJK, eccentricity of earth orbit e_Sun, sun eccentric anomaly E_Sun

Outputs : Earth Velocity magnitude and component Xw Yw in frame PQW in meter per sec

E17B 011. Velocity magnitude meter per sec using GM, SMA, r earth EC to SC frame IJK at UT time = 29290.9998931436

E17C 011. Velocity component meter per sec in orbit Xw using GM, e_Sun, SMA, E_Sun at UT time = -0.000001456

E17D 011. Velocity component meter per sec in orbit Yw using GM, e_Sun, SMA, E_Sun at UT time = -29290.9998931436

Finding Earth Velocity Vector [vX, vY, vZ] in meter per sec in orbit; a Transform of [Xw, Yw] in frame PQW To [vX, vY, vZ] in frame XYZ

Inputs : velocity component (Xw, Yw), sun right ascension Alpha, Sun Argument of perigee W_Sun, inclination Epcylone

Outputs : earth velocity vector(vX, vY, vZ, vR) meter per sec in frame XYZ

E17E 011. vX earth Velocity vector meter per sec using Xw Yw frame PQW RA w i at UT time = 1114.6970357569

E17F 011. vY earth Velocity vector meter per sec using Xw Yw frame PQW RA w i at UT time = -29149.2525417419

E17G 011. vZ earth Velocity vector meter per sec using Xw Yw frame PQW RA w i at UT time = 2653.5262418839

E17H 011. vR earth Velocity magnitude meter per sec using Xw Yw frame PQW RA w i at UT time = 29290.9998931436

Earth State Vectors : Position [X, Y, Z] in km and Velocity [Vx, Vy, Vz] in meter per sec, at time input UT.

18. Finding Earth State Position Vector [X, Y, Z] in km at time input UT.

Inputs : position vector(r_I, r_J, r_K, r) in frame IJK values assigned to state position vector

Outputs : State Position Vector(X, Y, Z, R) in km, frame XYZ

E18A 011. State vector position X km at UT time = -140226215.3950415300

E18B 011. State vector position Y km at UT time = 446.2552520869

E18C 011. State vector position Z km at UT time = 58911327.9440127610

E18D 011. State vector position R km at UT time = 152098441.9523840500

19. Finding Earth State Velocity Vector [Vx, Vy, Vz] in meter per sec at time input UT.

Inputs : velocity vector(v_X, v_Y, v_Z, v_R) meter per sec in frame XYZ values assigned to state velocity vector

Outputs : state velocity vector(Vx, Vy, Vz, V) meter per sec, frame XYZ

E19A 011. State vector velocity Vx meter per sec at UT time = 1114.6970357569

E19B 011. State vector velocity Vy meter per sec at UT time = -29149.2525417419

E19C 011. State vector velocity Vz meter per sec at UT time = 2653.5262418839

019D 011. State vector velocity V meter per sec at UT time = 29290.9998931436

20. Earth Orbit Normal Vector [Wx, Wy, Wz] in km and angles Delta, i, RA at time input UT; Normal is line perpendicular to orbit plane.

Inputs : earth pos r EC to SC frame IJK, inclination Epcylone, sun right ascension Alpha

Outputs : earth orbit normal vector (Wx, Wy, Wz, W) in km

E20A 011. Earth orbit normal W km using r earth pos frame IJK inclination Alpha = 152098441.9523840500

E20B 011. Earth orbit normal Wx km using r earth pos frame IJK inclination Alpha = 58626279.9571206120

E20C 011. Earth orbit normal Wy km using r earth pos frame IJK inclination Alpha = 14945281.0276021960

E20D 011. Earth orbit normal Wz km using r earth pos frame IJK inclination Alpha = 139547604.4858458000

020E 011. Earth orbit normal Delta W deg using r earth pos frame IJK inclination Alpha = 66.5607205617

E20F 011. Earth orbit normal Inclination i deg using normal_Delta_W = 23.4392794383

E20G 011. Earth orbit normal Alpha W deg using r earth pos frame IJK, inclination, Alpha = 14.3014954901

E20H 011. Earth orbit normal Right ascension of ascending node using normal Alpha, W = 104.3014954901

Transform Earth State Vectors to Earth position Keplerian elements.

21. Finding Earth position Keplerian elements computed using State Vector, at time input UT.

Inputs : State vector year, days decimal of year, revolution, node, State Position Vector [X, Y, Z], State Velocity Vector [Vx, Vy, Vz]

Outputs : Keplerian elements : year, days decimal of year, revolution, node, inclination, right ascension, eccentricity, argument of perigee, mean anomaly, mean motion rev per day, mean angular velocity rev per day, mean motion rev_per_day from SMA considering oblateness

E21A 011. Keplerian elements year = 2013, days_decimal_of_year = 185.01311, revolution no = 1, node = 1 ie ascending

E21B 011. inclination_deg = 23.4392794383

E21C 011. right ascension ascending node deg = 104.3014954901

E21D 011. eccentricity = 0.0167102190

E21E 011. argument of perigee_deg = 256.8356315327

E21F 011. mean anomaly deg = 179.9999999997

E21G 011. mean_motion rev per day = 0.0027377786

E21H 011. mean angular velocity rev_per_day = 0.0027377786

E21I 011. mean motion rev per day using SMA considering oblateness = 0.0027377786

Transform Earth position Keplerian elements to Earth State Vectors .

22. Finding Earth position State Vectors, computed using Keplerian elements at time input UT

(computed again to validate model equations, Keplerian elements to State Vectors & back)

Inputs : Keplerian elements : year, days decimal of year, revolution, node, inclination, right ascension, eccentricity, argument of perigee, mean anomaly, mean motion rev per day, mean angular velocity rev per day, mean motion rev_per_day from SMA considering oblateness

Outputs : State vector year, days decimal of year, revolution, node, State Position Vector [X, Y, Z], State Velocity Vector [Vx, Vy, Vz]

E22A 011. State vectors year = 2013, days_decimal_of_year = 185.01311, revolution no = 1, node = 1 ie decending

E22B 011. state vector position X km = -140226215.3950413500, state vector velocity Vx meter per sec = 1114.6970357568

E22C 011. state vector position Y km = 446.2552521827, state vector velocity Vy meter per sec = -29149.2525417419

E22D 011. state vector position Z km = 58911327.9440126870, state vector velocity Vz meter per sec = 2653.5262418839

E22E 011. state vector position R km = 152098441.9523838800, state vector velocity V meter per sec = 29290.9998931436

Note : Computation of all above parameters, grouped in 1 to 22, corresponds to time

(a) Universal time over Greenwich (UT/GMT) : Year = 2013, Month = 7, Day = 5, Hour = 0, Min = 18, Sec = 52.593

(b) Mean Solar time (MST) over Earth Observation point (EP) : Year = 2013, Month = 7, Day = 5, Hour = 5, Min = 28, Sec = 31.620

Move on to next Astronomical event in orbit Earth around Sun.

Next Section - 4.6 Position of earth at time when earth is at autumnal equinox

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Earth Positional Parameters on Celestial Sphere : Input Year Time when Earth is at Autumnal equinox .

6. Finding Position of Earth on Celestial Sphere at Input Universal Time, when Earth is at Autumnal equinox .

Input UT Time, when Earth is at Autumnal equinox : year = 2013, month = 9, day = 22, hour = 20, minute = 45, seconds = 38.50711
 Julian Day = 2456558.36503, year_day_decimal = 264.86503, day_hour_decimal = 20.76070

Observation Point on Earth (Bhopal, India) : Lat +ve or -ve 0 to 90 deg = 23.25993 ie deg = 23, min = 15, sec = 35.76
 Log 0 to 360 deg = 77.41261 ie deg = 77, min = 24, sec = 45.41
 Alt from earth surface in km = 0.49470

First Compute the **Sun Position** on Celestial Sphere, then Compute the **Earth Position** on Celestial Sphere.

(A) Computed Values for SUN POSITION on Celestial Sphere at Input Ut Time : (Sr. No 1 - 22)

01. Earth around Sun Mean motion rev per day (mm)	= 0.0027377786	02. Semi-major axis in km considering oblateness (SMA)	= 149598616.31172
03. Earth mean motion deg per day using SMA (mm)	= 0.9856003000	04. Sun mean movement deg per day (n sun)	= 0.9856003000
05. Eccentricity of earth orbit (e sun)	= 0.0167102190	06. Perihelion to input time diff in Julian days	= 262.4817348463
07. Mean anomaly in deg per day from n_sun (m sun)	= 258.7020766091	08. Sun Mean Longitude in deg (Lmean)	= 181.8702061020
09. Earth Mean anomaly in deg (ME)	= 258.7020766091	10. Sun Ecliptic Longitude in deg (Lsun)	= 180.0000000001
11. Obliquity of ecliptic in deg (Epcylone)	= 23.4374939503	12. Sun Right ascension in deg (Alpha)	= 180.0000000001
13. Sun Declination in deg (Delta)	= -0.0000000001	14. Sun Mean distance in km (As)	= 149598616.31172
15. Sun Radial distance from earth in km (Rs)	= 150128632.16764	16. Sun Nodal elongation in deg (U sun)	= -0.0000000001
17. Sun Mean anomaly in deg (M sun)	= 258.7020766085	18. Sun Eccentric anomaly in deg (E sun)	= 257.7663930098
19. Sun True anomaly in deg (T sun)	= 256.8323186392	20. Sun Argument of perigee in deg (W sun)	= 103.1676813607
21. Sun True anomaly in deg from U & W (V sun)	= 256.8323186392	22. Sun Distance in km (d sun)	= 150048057.36583

Sun Ecliptic latitude is always nearly zero (the value never exceeds 0.00033 deg)

These Values are applied as input for Computing Earth Position on Celestial Sphere around Sun at same input UT Time.

(B) Computed Values for EARTH POSITION on Celestial Sphere around Sun at same Input Ut Time : (Sr. No 1 - 22)

Input Time year = 2013, month = 9, day = 22, hour = 20, minute = 45, seconds = 38.50711, corresponding Julian Day = 2456558.3650290174
 Observation Point on Earth : Lat +ve or -ve 0 to 90 deg = 23.25993, Log 0 to 360 deg = 77.41261, Alt from earth surface in km = 0.49470
 Sun position on Celestial sphere at input time, computed above total 22 parameters.

Output Earth Position on Celestial sphere around Sun : **Computed below around 120 parameters, presented in 1-22 groups.**
 Number is large, because some parameters are computed using more than one model equation, that require different inputs.
 This helps in validation of results and understanding the different input considerations.

01. Finding GST Greenwich sidereal time and GHA Greenwich hour angle in 0 to 360 deg, at input UT time YY MM DD HH.

Note - for GST, the year 1900 JAN day_1 hr 1200 is ref for time difference in terms of julian_century,
 for GHA, the year 2000_JAN_day_1 hr_1200 is ref for time difference in terms of julian days.

Inputs : Time UT year = 2013, month = 9, day = 22, hour = 20, minute = 45, seconds = 38.50711

Outputs : GST & GHA in 0-360 deg over Greenwich.

E01A 011. GST Greenwich sidereal time in 0-360 deg, over Greenwich = 313.28074, hr = 20, min = 53, sec = 7.37817

E01B 011. GHA Greenwich hour angle in 0 to 360 deg, over Greenwich = 313.29307, deg = 313, min = 17, sec = 35.04994

02. Finding Earth latitude & longitude pointing to Sun Ecliptic longitude(Lsun).

Inputs : earth inclination, sun true anomaly T_Sun, sun argument of perigee W_Sun, sun right ascension Alpha,
 earth equator radius, GST at input UT, log SS & EP, earth orbit radius EC to SC

Outputs : Earth lat & log pointing to Lsun.

E02A 011. Earth latitude +ve or -ve in 0 to 90 deg at UT time = -0.00 ie deg = 0, min = 0, sec = 0.00

E02B 011. Earth longitude 0 to 360 deg = 226.72 ie deg = 226, min = 43, sec = 9.33

03. Finding LST over three longitudes, Greenwich log, Sun mean log (Lmean), and Sun ecliptic log (Lsun) .

Note - for LST, used sidereal time at Greenwich GST and desired geographic longitude

Inputs : At Time input UT - GST, Log of Greenwich, sun mean log Lmean, Sun ecliptic log Lsun.

Outputs : LST over Greenwich, Lmean, Lsun .

E03A 011. LST Local sidereal time in 0-360 deg, over Greenwich longitude = 313.28074, hr = 20, min = 53, sec = 7.37817

E03B 011. LST Local sidereal time in 0-360 deg, over Sun mean longitude (Lmean) = 135.15095, hr = 9, min = 0, sec = 36.22763
 E03C 011. LST Local sidereal time in 0-360 deg, over Sun epliptic longitude (Lsun) = 133.28074, hr = 8, min = 53, sec = 7.37817

04. Finding ST0 sidereal time over Greenwich longitude = 0.0, at time input Year JAN day 1 hr 00.

Note - this is sidereal time ST at UT year, month = 1, day = 1, hours decimal = 0.0 and geographic longitude = 0.0

Inputs : Time input UT Year, JAN day 1 hr 00, Log 0.0

Outputs : ST0 over Greenwich

E04 011. ST0 Sidereal time in 0-360 deg, over Greenwich at input UT year, MM 1, DD 1, HH 00 = 100.80678, hr = 6, min = 43, sec = 13.62710

05. Finding ST sidereal time over three longitudes of, Greenwich log, Sun mean log (Lmean), and Sun epliptic log (Lsun) .

Note - this is local sidereal time LST; (LST = GST at UT time + geographic longitude).

Inputs : At Time input UT - Log 0.0, Log Lmean, Log Lsun

Outputs : ST over Greenwich, Lmean, Lsun.

E05A 011. ST Sidereal time in 0-360 deg, over Greenwich at input UT time = 313.28074, hr = 20, min = 53, sec = 7.37817

E05B 011. ST Sidereal time in 0-360 deg, over Sun mean longitude (Lmean) at input UT time = 135.15095, hr = 9, min = 0, sec = 36.22763

005C 011. ST Sidereal time in 0-360 deg, over Sun longitude (Lsun) at input UT time = 133.28074, hr = 8, min = 53, sec = 7.37817

06. Finding H hour angle in 0 to 360 deg over longitudes of, Greenwich, Lmean, Lsun, Earth Sub Sun point SS, Earth Obseration point EP.

Note - used Sun Right ascension Alpha at input time; (hour angle HA = LST - Alpha).

Inputs : At Time input UT - Sun Right ascension Alpha and ST Sidereal time over longitudes 0.0, Lmean, Lsun, SS, EP

Outputs : Hour Angles over Greenwich, Lmean, Lsun, SS, EP

E06A 011. H hour angle 0-360 deg, over Greenwich, = 133.28074, deg = 133, min = 16, sec = 50.67250

E06B 011. H hour angle 0-360 deg, over Lmean, = 315.15095, deg = 315, min = 9, sec = 3.41447

E06C 011. H hour angle 0-360 deg, over Lsun, = 313.28074, deg = 313, min = 16, sec = 50.67250

E06D 011. H hour angle 0-360 deg, over SS, = 0.00000, deg = 0, min = 0, sec = 0.00000

E06E 011. H hour angle 0-360 deg, over EP, = 210.69336, deg = 210, min = 41, sec = 36.08650

07. Finding Delta E is Equation of Time in seconds, at time input UT.

Note - this value in seconds accounts for relative movement of sun in elliptical orbit w.r.t earth and effect of obliquity of the ecliptic;

its maximum value is 16 minutes (960 sec.); Delta E is computed using time in days from the perihelion, n_sun_deg and w_sun at input UT.

Inputs : Time input UT in JD, time perihelion in JD, Sun mean movement n_sun, Eccentricity of earth orbit E_Sun

Outputs : Delta E time_equation in seconds.

E07 011. Delta E Time Equation in seconds = -489.69657, hr = 0, min = 8, sec = 9.69657

08. Finding GST Greenwich sidereal time, and GHA Greenwich hour angle 0 to 360 deg at time when earth is at perihelion.

Inputs : Time in JD when earth at perihelion YY = 2013, MM = 1, DD = 3, hr = 9, min = 11, sec = 56.62

Outputs : GST & GHA in 0-360 deg over Greenwich when earth is at perihelion

E08A 011. GST sidereal time in 0-360 deg over Greenwich at time when earth is at perihelion = 241.14177, hr = 16, min = 4, sec = 34.02452

E08B 011. GHA hour angle in 0-360 deg over Greenwich at time when earth is at perihelion = 241.14717, hr = 16, min = 4, sec = 35.32064

09. Finding ST sidereal time and MST mean sidereal time, over Greenwich, using Earth mean motion rev per day .

Inputs : GST when earth at perihelion, earth rotation rate, ref. JD2000, time input UT in JD, time perihelion in JD.

Outputs : STP, angle perihelion to input JD, ST over Greenwich, MSTO & MST over Greenwich, solar time

E09A 011. STP sidereal time in 0-360 deg over Greenwich when earth at perihelion = 241.14177, hr = 16, min = 4, sec = 34.02452

E09B 011. Angle in 0-360 deg from earth at perihelion to input JD using earth rotational rate = 72.12977,

E09C 011. ST in 0-360 deg over Greenwich using STP and angle from perihelion at input JD = 313.27154, hr = 20, min = 53, sec = 5.16880

E09D 011. ST in 0-360 deg over Greenwich using STP and earth rotation at UT time = 313.27154, hr = 20, min = 53, sec = 5.16880

E09E 011. MSTO in deg, over Greenwich using JD century days, ref J2000 to I/P YY, M1, D1 hr 00 = 100.80714, hr = 6, min = 43, sec = 13.71450

E09F 011. MST in deg, over Greenwich using JD century in days from ref J2000 to UT time Y M D H = 313.28111, hr = 20, min = 53, sec = 7.46617

E09G 011. Solar time over Greenwich in JD (GMT or input UT - 12 hr) = YY 2013, MM 9, DD 22, hr 8, min 45, sec 38.507, ie JD 2456557.86503

10. Finding Earth orbit radius using true anomaly, Sub Sun point (SS) on earth surface and related parameters .

(a) Finding Earth orbit radius using true anomaly.

Inputs : semi-major axis SMA, eccentricity of earth orbit e_sun, sun true anomaly T_Sun

Outputs : earth orbital radius EC to SC (earth center to sun center)

E10A 011. earth orbital radius EC to SC km using true anomaly at UT time = 150128324.5301490100

(b) Finding Sub Sun point (SS) over earth surface (Latitude, Longitude, & Latitude radius) pointing to Sun Ecliptic Log (Lsun), Sun height from earth surface over SS, and LST over SS log at time input UT.

Note - for SS Latitude, used earth inclination, sun true anomaly T_sun and sun argument of perigee w_sun.
for SS Longitude, used Sun right ascension Alpha and sidereal time at Greenwich GST.

Inputs : earth inclination, sun true anomaly T_Sun, sun argument of perigee W_Sun, sun right ascension Alpha, earth equator radius, GST at input UT, log SS & EP, earth orbit radius EC to SC

Outputs : SS point Latitude, Longitude, Latitude radius, LST & LMT over SS .

E10B 011. SS point Latitude +ve or -ve in 0 to 90 deg at UT time = -0.00 ie deg = 0, min = 0, sec = 0.00

E10C 011. SS point Longitude 0 to 360 deg = 226.72 ie deg = 226, min = 43, sec = 9.33

E10D 011. SS point Latitude radius km at UT time = 6378.1440000000

E10E 011. Sun height km from earth surface over SS at UT time = 150121946.3861490200

E10F 011. LST local sidereal time in 0-360 deg over SS log at UT time, (LST = GST + log east) = 180.000 ie hr = 12, min = 0, sec = 0.08799
LST local sidereal time and LMT local mean time with date adjusted to calendar YY MM DD and UT hr mm sec.

E10G 011. LST local sidereal time at Sub Sun point (SS) YY = 2013, MM = 9, DD = 22, hr = 12, min = 0, sec = 0.09

E10H 011. LMT local Mean time at Sub Sun point (SS) YY = 2013, MM = 9, DD = 22, hr = 11, min = 52, sec = 31.13

(c) Finding LST and LMT over Earth point(EP) where Observer is, at time input UT.

Inputs : EP point Latitude, Longitude

Outputs : LST & LMT over EP .

E10I 011. LST local sidereal time in 0-360 deg at EP log at UT time, (LST = GST + log east) = 30.694 ie hr = 2, min = 2, sec = 46.49378
LST and LMT with date adjusted to calendar YY MM DD and UT hr mm sec.

E10J 011. LST local sidereal time at Earth point (EP) YY = 2013, MM = 9, DD = 23, hr = 2, min = 2, sec = 46.49

E10K 011. LMT local Mean time at Earth point (EP) YY = 2013, MM = 9, DD = 23, hr = 1, min = 55, sec = 17.53

Finding Earth to Sun Position Vectors coordinate in PQW, IJK, SEZ frames and the Vector Coordinate Transforms.

First defined coordinate systems, PQW, IJK, SEZ, then computed Position & Velocity vectors in these three coordinate systems.

(a) **Perifocal Coordinate System (POW)**, is Earth Centered Inertial coordinate frame defined in terms of Kepler Orbital Elements.

The system is fixed with time (inertial), pointing towards orbit periapsis;

the system's origin is Earth center (EC), and its fundamental plane is the orbit plane;

the P-vector axis directed from EC toward the periapsis of the elliptical orbit plane,

the Q-vector axis sweeps 90 deg from P axis in the direction of the orbit,

the W-vector axis directed from EC in a direction normal to orbit plane, forms a right-handed coordinate system.

(b) **Geocentric Coordinate System (IJK)**, is also an Earth Centered Inertial (ECI) frame, a Conventional Inertial System (CIS).

The system is fixed with time (inertial), pointing towards vernal equinox;

the system's origin is Earth center (EC), and its fundamental plane is the equator;

the I-vector is +X-axis directed towards the vernal equinox direction on J2000, Jan 1, hr 12.00 noon,

the J-vector is +Y-axis sweeps 90 deg to the east in the equatorial plane,

the K-vector is +Z-axis directed towards the North Pole.

(c) **Topocentric Horizon Coordinate System (SEZ)**, is Non-Inertial coordinate frame, known as Earth-Centered Earth-Fixed Coordinates (ECEF).

The system moves with earth, is not fixed with time (non-inertial), is for use by observers on the surface of earth;

the observer's surface forms the fundamental plane, is tangent to earth's surface

the S-vector is +ve horizontal -axis directed towards South,

the E-vector is +ve horizontal -axis directed towards East,

the Z-vector is +ve normal directed upwards on earth surface.

Note that axis Z not necessarily pass through earth center, so not used to define as radius vector.

11. Finding Earth center(EC) to Sun center(SC) Range Vector[r_p , r_q , r] from in PQW frame, perifocal coordinate system.

Inputs : Semi-major axis (SMA), Eccentricity of earth orbit (e_{sun}), Sun eccentric anomaly (E_{sun})

Outputs : Vector(r , r_p r_q) in PQW frame

E11A 011. r earth pos vector magnitude EC to SC km in PQW frame perifocal cord at UT time = 150128324.53015

E11B 011. rp earth pos vector component EC to SC km in PQW frame perifocal cord at UT time = -34199483.1036060300

E11C 011. rq earth pos vector component EC to SC km in PQW frame perifocal cord at UT time = -146181083.5288748700

Note - r earth pos vector magnitude EC to SC km in PQW frame is same as earth orbital radius computed before using true anomaly.

12. Transform_1 Earth position EC to SC Range Vector[r_p , r_q] in PQW frame To Range Vector[r_I , r_J , r_K] in IJK frame, inertial system cord.

Inputs : Vector(r_p , r_q) EC_to_SC km in frame PQW , Alpha rd, w_{sun} rd, earth_inclination rd ,

Outputs : Vector(r_I , r_J , r_K , r) EC_to_SC km in frame IJK

E12A 011. r_I earth pos vector component EC to SC km frame IJK at UT time = -150128324.5301489500

E12B 011. r_J earth pos vector component EC to SC km frame IJK at UT time = 0.0000000075

E12C 011. r_K earth pos vector component EC to SC km frame IJK at UT time = -0.0001395773

E12D 011. r earth pos vector magnitude EC to SC km frame IJK at UT time = 150128324.5301489500

Note - r earth pos vector magnitude EC to SC km in PQW frame is same as that computed above in PQW frame.

13. Transform_2 Earth point EP(lat, log, hgt) To EC to SC Range Vector[R_I , R_J , R_K , R] in IJK frame.

Inputs : earth equator radius_km, earth point EP(lat deg, log deg, hgt meter),

LST_local_sidereal_time_in_0_to_360_deg_at_EP_log_at_UT_time,

Outputs : Vector(R_I , R_J , R_K , R) Range EC to EP in IJK frame

E13A 011. R_I pos vector component EC to EP km frame IJK at UT time = 5036.6074245797

E13B 011. R_J pos vector component EC to EP km frame IJK at UT time = 2989.7725118111

E13C 011. R_K pos vector component EC to EP km frame IJK at UT time = 2517.6312937817

E13D 011. R pos vector magnitude EC to EP km frame IJK at UT time = 6375.3134317570

14. Transform_3 Earth position EC to SC Range Vectors [r_I r_J r_K] & [R_I R_J R_K] To EP to SC Range Vector[r_{vI} , r_{vJ} , r_{vK}] in IJK frame.

Inputs : Vector(r_I r_J r_K) position EC_to_SC km in frame IJK , Vector(R_I R_J R_K) range EC to SC km in IJK frame,

Outputs : Vector(r_{vI} , r_{vJ} , r_{vK} , r_v) range EP to SC in IJK frame

E14A 011. r_{vI} range vector component EP to SC km frame IJK at UT time = -150133361.1375735400

E14B 011. r_{vJ} range vector component EP to SC km frame IJK at UT time = -2989.7725118037

E14C 011. r_{vK} range vector component EP to SC km frame IJK at UT time = -2517.6314333590

E14D 011. r_v range vector magnitude EP to SC km frame IJK at UT time = 150133361.1884523300

15. Transform_4 Earth point EP to SC Range Vector[rvI, rvJ, rvK] in IJK frame To EP to SC Range Vector[rvS, rvE, rvZ] in SEZ frame.

Inputs : lat_pos_neg_0_to_90_deg_at_EP_at_time_UT , LST_local_sidereal_time_in_0_to_360_deg_at_EP_log_at_UT_time

Vector(rvI, rvJ, rvK, rv) range EP to SC km in IJK frame,

Outputs : Vector(rvS, rvE, rvZ, rv) range EP to SC km in SEZ frame

E15A 011. rvS range vector component EP to SC km frame SEZ at UT time = -50980643.1144296600

E15B 011. rvE range vector component EP to SC km frame SEZ at UT time = 76632812.6748455610

E15C 011. rvZ range vector component EP to SC km frame SEZ at UT time = -118610337.6225001200

E15D 011. rv range vector magnitude EP to SC km frame SEZ at UT time = 150133361.1884523300

16. Finding Elevation(EL) and Azimuth(AZ) angle of Sun at Earth Observation point EP .

Note : Results **computed using 4 different formulations**, each require different inputs to give EL & AZ angles.

For all situations of Object and Observer positions, a combination of Latitude N/S & Longitude E/W :

Method 1 : for both EL & AZ angles, this does not provide correct results ;

Method 2 : for only EL angle, this provides consistent, unambiguous correct results.

but for AZ angles the results are ambiguous, need corrections by adding or subtracting values as 180 or 360 or sign change.

Method 3 : same as method 2, for EL angle, the results are correct, but for AZ angles the results are ambiguous, need corrections.

Method 4 : for finding Azimuth and Distance but not for finding Elevation angle;

for AZ angles, this provides correct unambiguous results that need no further corrections.

Therefore for Elevation (EL) angle Method 3 results are accepted and for Azimuth (AZ) angle Method 4 results are accepted .

Results verified from other sources; Ref URLs <http://www.ga.gov.au/geodesy/astro/smpos.jsp#intzone> .

NOAA Research <http://www.esrl.noaa.gov/gmd/grad/solcalc/> , and <http://aa.usno.navy.mil/data/docs/AltAz.php>

Xavier Jubier, Member IAU http://xjubier.free.fr/en/site_pages/astronomy/ephemerides.html

Rem: SS point lat deg = -0.00, log deg = 226.72 YY = 2013, MM = 9, DD = 22, hr = 11, min = 52, sec = 31.13

EP point lat deg = 23.26, log deg = 77.41 YY = 2013, MM = 9, DD = 23, hr = 1, min = 55, sec = 17.53

Elevation(EL) & Azimuth(AZ) angle of Sun at Earth Observation point EP, (**method 1** - computed values may be Ambiguous or Incorrect).

Inputs : Vector[rvS, rvE, rvZ] range EP to SC km in SEZ frame

Outputs : Elevation(EL) & Azimuth(AZ) of Sun at EP

E16A 011. Elevation angle deg of Sun at EP using rv SEZ at UT time = -52.1886128588

E16B 011. Azimuth angle deg of sun at EP using rv SEZ at UT time = 56.3658187650

Elevation(EL) & Azimuth(AZ) angle of Sun at Earth Observation point EP, (method 2 - computed AZ values may be ambiguous & incorrect).

Inputs : Time input UT YY MM DD HH, Equator radius, EP lat & log, SS lat & log, Sun declination Delta

Outputs : Elevation(EL) & Azimuth(AZ) of Sun at EP

E16C 011. Elevation angle deg of Sun at EP using Sun declination diff log range EP to SC = -52.18740

E16D 011. Azimuth angle deg of sun at EP using sun declination diff log range EP to SC = 33.63457

Elevation(EL) & Azimuth(AZ) angle of Sun at Earth Observation point EP, (method 3 - computed AZ values may be ambiguous & incorrect).

Inputs : Time input UT YY MM DD HH, Equator radius, EP lat & log, SS lat & log, Sun hgt from EC, Sun range from EP

(Sun hgt from EC = earth orbit radius EC to SC km ; Sun range from EP = rv range vector EP to SC km frame SEZ)

Outputs : Elevation(EL) & Azimuth(AZ) of Sun at EP

E16E 011. Elevation angle deg of Sun at EP using Sun hgt diff log range EP to SC = -52.18889 ie deg = -52, min = 11, sec = 20.02

E16F 011. Azimuth angle deg of sun at EP using sun hgt diff log range EP to SC = 236.36543 ie deg = 236, min = 21, sec = 55.56

Azimuth(AZ) angle of Sun at Earth Observation point EP, (method 4 - computed AZ values is unambiguous & correct).

Inputs : Time input UT YY MM DD HH, EP lat & log, SS lat & log

Outputs : Azimuth(AZ) of Sun at EP

E16G 011. Azimuth angle deg of sun at EP using sun hgt diff log range EP to SC = 56.36543 ie deg = 56, min = 21, sec = 55.56

Due to such incorrect results, finally for Elevation (EL) Method 3 results and for Azimuth (AZ) Method 4 results are accepted.

Finally accepted Elevation angle deg of Sun from EP to SC = -52.1888941383 ie deg = -52, min = 11, sec = 20.02

Finally accepted Azimuth angle_deg of Sun from EP to SC = 56.3654334756 ie deg = 56, min = 21, sec = 55.56

Distance in km from Earth observation point(EP) to Sub Sun point(SS) and Earth Velocity meter per sec in orbit at time input UT.

17. Finding Distance in km from Earth observation point(EP) to Sub Sun point(SS) over Earth surface .

Inputs : EP lat & log, SS lat & log,

Outputs : Distance in km from EP to SS over Earth surface

E17A 011. Distance in km Earth observation point(EP) to Sub Sun point(SS) = 15828.20917

Finding Earth Velocity meter per sec in orbit in frame PQW

Inputs : semi-major axis SMA, GM_Sun, earth pos r EC to SC frame IJK, eccentricity of earth orbit e_Sun, sun eccentric anomaly E_Sun

Outputs : Earth Velocity magnitude and component Xw Yw in frame PQW in meter per sec

E17B 011. Velocity magnitude meter per sec using GM, SMA, r earth EC to SC frame IJK at UT time = 29679.3404099639

E17C 011. Velocity component meter per sec in orbit Xw using GM, e_Sun, SMA, E_Sun at UT time = 29005.5570441560

E17D 011. Velocity component meter per sec in orbit Yw using GM, e_Sun, SMA, E_Sun at UT time = -6288.1561469737

Finding Earth Velocity Vector [vX, vY, vZ] in meter per sec in orbit; a Transform of [Xw, Yw] in frame PQW To [vX, vY, vZ] in frame XYZ

Inputs : velocity component (Xw, Yw), sun right ascension Alpha, Sun Argument of perigee W_Sun, inclination Epcylone

Outputs : earth velocity vector(vX, vY, vZ, vR) meter per sec in frame XYZ

E17E 011. vX earth Velocity vector meter per sec using Xw Yw frame PQW RA w i at UT time = 484.6892104138

E17F 011. vY earth Velocity vector meter per sec using Xw Yw frame PQW RA w i at UT time = -27226.6334875568

E17G 011. vZ earth Velocity vector meter per sec using Xw Yw frame PQW RA w i at UT time = 11804.1836852055

E17H 011. vR earth Velocity magnitude meter per sec using Xw Yw frame PQW RA w i at UT time = 29679.3404099639

Earth State Vectors : Position [X, Y, Z] in km and Velocity [Vx, Vy, Vz] in meter per sec, at time input UT.

18. Finding Earth State Position Vector [X, Y, Z] in km at time input UT.

Inputs : position vector(r_I, r_J, r_K, r) in frame IJK values assigned to state position vector

Outputs : State Position Vector(X, Y, Z, R) in km, frame XYZ

- E18A 011. State vector position X km at UT time = -150128324.5301489500
- E18B 011. State vector position Y km at UT time = 0.0000000075
- E18C 011. State vector position Z km at UT time = -0.0001395773
- E18D 011. State vector position R km at UT time = 150128324.5301489500

19. Finding Earth State Velocity Vector [Vx, Vy, Vz] in meter per sec at time input UT.

Inputs : velocity vector(v_X, v_Y, v_Z, v_R) meter per sec in frame XYZ values assigned to state velocity vector

Outputs : state velocity vector(Vx, Vy, Vz, V) meter per sec, frame XYZ

- E19A 011. State vector velocity Vx meter per sec at UT time = 484.6892104138
- E19B 011. State vector velocity Vy meter per sec at UT time = -27226.6334875568
- E19C 011. State vector velocity Vz meter per sec at UT time = 11804.1836852055
- 019D 011. State vector velocity V meter per sec at UT time = 29679.3404099639

20. Earth Orbit Normal Vector [Wx, Wy, Wz] in km and angles Delta, i, RA at time input UT; Normal is line perpendicular to orbit plane.

Inputs : earth pos r EC to SC frame IJK, inclination Epcylone, sun right ascension Alpha

Outputs : earth orbit normal vector (Wx, Wy, Wz, W) in km

- E20A 011. Earth orbit normal W km using r earth pos frame IJK inclination Alpha = 150128324.5301489500
- E20B 011. Earth orbit normal Wx km using r earth pos frame IJK inclination Alpha = -0.0001280575
- E20C 011. Earth orbit normal Wy km using r earth pos frame IJK inclination Alpha = 59717589.8947850020
- E20D 011. Earth orbit normal Wz km using r earth pos frame IJK inclination Alpha = 137740056.9311193500
- 020E 011. Earth orbit normal Delta W deg using r earth pos frame IJK inclination Alpha = 66.5607205617
- E20F 011. Earth orbit normal Inclination i deg using normal_Delta_W = 23.4392794383
- E20G 011. Earth orbit normal Alpha W deg using r earth pos frame IJK, inclination, Alpha = -89.9999999999
- E20H 011. Earth orbit normal Right ascension of ascending node using normal Alpha, W = 0.0000000001

Transform Earth State Vectors to Earth position Keplerian elements.

21. Finding Earth position Keplerian elements computed using State Vector, at time input UT.

Inputs : State vector year, days decimal of year, revolution, node, State Position Vector [X, Y, Z], State Velocity Vector [Vx, Vy, Vz]

Outputs : Keplerian elements : year, days decimal of year, revolution, node, inclination, right ascension, eccentricity, argument of perigee, mean anomaly, mean motion rev per day, mean angular velocity rev per day, mean motion rev_per_day from SMA considering oblateness

E21A 011. Keplerian elements year = 2013, days_decimal_of_year = 264.86503, revolution no = 1, node = 2 ie decending

E21B 011. inclination_deg = 23.4392794383

E21C 011. right ascension ascending node deg = 180.000000001

E21D 011. eccentricity = 0.0167102190

E21E 011. argument of perigee_deg = 103.1676813607

E21F 011. mean anomaly deg = 258.7020766085

E21G 011. mean_motion rev per day = 0.0027377786

E21H 011. mean angular velocity rev_per_day = 0.0027377786

E21I 011. mean motion rev per day using SMA considering oblateness = 0.0027377786

Transform Earth position Keplerian elements to Earth State Vectors .

22. Finding Earth position State Vectors, computed using Keplerian elements at time input UT

(computed again to validate model equations, Keplerian elements to State Vectors & back)

Inputs : Keplerian elements : year, days decimal of year, revolution, node, inclination, right ascension, eccentricity, argument of perigee, mean anomaly, mean motion rev per day, mean angular velocity rev per day, mean motion rev_per_day from SMA considering oblateness

Outputs : State vector year, days decimal of year, revolution, node, State Position Vector [X, Y, Z], State Velocity Vector [Vx, Vy, Vz]

E22A 011. State vectors year = 2013, days_decimal_of_year = 264.86503, revolution no = 1, node = 2 ie decending

E22B 011. state vector position X km = -150128324.5301487700, state vector velocity Vx meter per sec = 484.6892104141

E22C 011. state vector position Y km = -0.0000012219, state vector velocity Vy meter per sec = -27226.6334875568

E22D 011. state vector position Z km = -0.0001390446, state vector velocity Vz meter per sec = 11804.1836852055

E22E 011. state vector position R km = 150128324.5301487700, state vector velocity V meter per sec = 29679.3404099639

Note : Computation of all above parameters, grouped in 1 to 22, corresponds to time

(a) Universal time over Greenwich (UT/GMT) : Year = 2013, Month = 9, Day = 22, Hour = 20, Min = 45, Sec = 38.507

(b) Mean Solar time (MST) over Earth Observation point (EP) : Year = 2013, Month = 9, Day = 23, Hour = 1, Min = 55, Sec = 17.535

Move on to next Astronomical event in orbit Earth around Sun.

Next Section - 4.7 Position of earth at time when earth is at winter solstice

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Earth Positional Parameters on Celestial Sphere : Input Year Time when Earth is at Winter solstice .

7. Finding Position of Earth on Celestial Sphere at Input Universal Time, when Earth is at Winter solstice .

Input UT Time, when Earth is at Winter solstice : year = 2013, month = 12, day = 21, hour = 17, minute = 10, seconds = 3.73442
 Julian Day = 2456648.21532, year_day_decimal = 354.71532, day_hour_decimal = 17.16770
Observation Point on Earth (Bhopal, India) : Lat +ve or -ve 0 to 90 deg = 23.25993 ie deg = 23, min = 15, sec = 35.76
 Log 0 to 360 deg = 77.41261 ie deg = 77, min = 24, sec = 45.41
 Alt from earth surface in km = 0.49470

First Compute the **Sun Position** on Celestial Sphere, then Compute the **Earth Position** on Celestial Sphere.

(A) Computed Values for SUN POSITION on Celestial Sphere at Input Ut Time : (Sr. No 1 - 22)

01. Earth around Sun Mean motion rev per day (mm)	=	0.0027377786	02. Semi-major axis in km considering oblateness (SMA)	=	149598616.31172
03. Earth mean motion deg per day using SMA (mm)	=	0.9856003000	04. Sun mean movement deg per day (n sun)	=	0.9856003000
05. Eccentricity of earth orbit (e sun)	=	0.0167102190	06. Perihelion to input time diff in Julian days	=	352.3320268290
07. Mean anomaly in deg per day from n_sun (m sun)	=	347.2585513424	08. Sun Mean Longitude in deg (Lmean)	=	270.4309127840
09. Earth Mean anomaly in deg (ME)	=	347.2585513424	10. Sun Ecliptic Longitude in deg (Lsun)	=	269.9999511320
11. Obliquity of ecliptic in deg (Epcylone)	=	23.4374619456	12. Sun Right ascension in deg (Alpha)	=	269.9999467376
13. Sun Declination in deg (Delta)	=	-23.4374619456	14. Sun Mean distance in km (As)	=	149598616.31172
15. Sun Radial distance from earth in km (Rs)	=	147162417.75585	16. Sun Nodal elongation in deg (U sun)	=	-90.0000000000
17. Sun Mean anomaly in deg (M sun)	=	347.2585513416	18. Sun Eccentric anomaly in deg (E sun)	=	347.0438922873
19. Sun True anomaly in deg (T sun)	=	346.8274560675	20. Sun Argument of perigee in deg (W sun)	=	283.1725439325
21. Sun True anomaly in deg from U & W (V sun)	=	346.8274560675	22. Sun Distance in km (d sun)	=	147158348.89183

Sun Ecliptic latitude is always nearly zero (the value never exceeds 0.00033 deg)

These Values are applied as input for Computing Earth Position on Celestial Sphere around Sun at same input UT Time.

(B) Computed Values for EARTH POSITION on Celestial Sphere around Sun at same Input Ut Time : (Sr. No 1 - 22)

Input Time year = 2013, month = 12, day = 21, hour = 17, minute = 10, seconds = 3.73442, corresponding Julian Day = 2456648.2153210002
 Observation Point on Earth : Lat +ve or -ve 0 to 90 deg = 23.25993, Log 0 to 360 deg = 77.41261, Alt from earth surface in km = 0.49470
 Sun position on Celestial sphere at input time, computed above total 22 parameters.

Output Earth Position on Celestial sphere around Sun : **Computed below around 120 parameters, presented in 1-22 groups.**
 Number is large, because some parameters are computed using more than one model equation, that require different inputs.
 This helps in validation of results and understanding the different input considerations.

01. Finding GST Greenwich sidereal time and GHA Greenwich hour angle in 0 to 360 deg, at input UT time YY MM DD HH.

Note - for GST, the year 1900 JAN day_1 hr 1200 is ref for time difference in terms of julian_century,
 for GHA, the year 2000_JAN_day_1 hr_1200 is ref for time difference in terms of julian days.

Inputs : Time UT year = 2013, month = 12, day = 21, hour = 17, minute = 10, seconds = 3.73442

Outputs : GST & GHA in 0-360 deg over Greenwich.

E01A 011. GST Greenwich sidereal time in 0-360 deg, over Greenwich = 347.94656, hr = 23, min = 11, sec = 47.17421
 E01B 011. GHA Greenwich hour angle in 0 to 360 deg, over Greenwich = 347.95673, deg = 347, min = 57, sec = 24.23424

02. Finding Earth Latitude & Longitude pointing to Sun Ecliptic Longitude(Lsun).

Inputs : earth inclination, sun true anomaly T_Sun, sun argument of perigee W_Sun, sun right ascension Alpha,
 earth equator radius, GST at input UT, log SS & EP, earth orbit radius EC to SC

Outputs : Earth Lat & Log pointing to Lsun.

E02A 011. Earth Latitude +ve or -ve in 0 to 90 deg at UT time = -23.44 ie deg = -23, min = 26, sec = 21.41
 E02B 011. Earth Longitude 0 to 360 deg = 282.05 ie deg = 282, min = 3, sec = 12.20

03. Finding LST over three longitudes, Greenwich log, Sun mean log (Lmean), and Sun ecliptic log (Lsun) .

Note - for LST, used sidereal time at Greenwich GST and desired geographic longitude

Inputs : At Time input UT - GST, Log of Greenwich, sun mean log Lmean, Sun ecliptic log Lsun.

Outputs : LST over Greenwich, Lmean, Lsun .

E03A 011. LST Local sidereal time in 0-360 deg, over Greenwich longitude = 347.94656, hr = 23, min = 11, sec = 47.17421

E03B 011. LST Local sidereal time in 0-360 deg, over Sun mean longitude (Lmean) = 258.37747, hr = 17, min = 13, sec = 30.59328
 E03C 011. LST Local sidereal time in 0-360 deg, over Sun epliptic longitude (Lsun) = 257.94651, hr = 17, min = 11, sec = 47.16248

04. Finding ST0 sidereal time over Greenwich longitude = 0.0, at time input Year JAN day 1 hr 00.

Note - this is sidereal time ST at UT year, month = 1, day = 1, hours decimal = 0.0 and geographic longitude = 0.0

Inputs : Time input UT Year, JAN day 1 hr 00, Log 0.0

Outputs : ST0 over Greenwich

E04 011. ST0 Sidereal time in 0-360 deg, over Greenwich at input UT year, MM 1, DD 1, HH 00 = 100.80678, hr = 6, min = 43, sec = 13.62710

05. Finding ST sidereal time over three longitudes of, Greenwich log, Sun mean log (Lmean), and Sun epliptic log (Lsun) .

Note - this is local sidereal time LST; (LST = GST at UT time + geographic longitude).

Inputs : At Time input UT - Log 0.0, Log Lmean, Log Lsun

Outputs : ST over Greenwich, Lmean, Lsun.

E05A 011. ST Sidereal time in 0-360 deg, over Greenwich at input UT time = 347.94656, hr = 23, min = 11, sec = 47.17421

E05B 011. ST Sidereal time in 0-360 deg, over Sun mean longitude (Lmean) at input UT time = 258.37747, hr = 17, min = 13, sec = 30.59328

005C 011. ST Sidereal time in 0-360 deg, over Sun longitude (Lsun) at input UT time = 257.94651, hr = 17, min = 11, sec = 47.16248

06. Finding H hour angle in 0 to 360 deg over longitudes of, Greenwich, Lmean, Lsun, Earth Sub Sun point SS, Earth Obseration point EP.

Note - used Sun Right ascension Alpha at input time; (hour angle HA = LST - Alpha).

Inputs : At Time input UT - Sun Right ascension Alpha and ST Sidereal time over longitudes 0.0, Lmean, Lsun, SS, EP

Outputs : Hour Angles over Greenwich, Lmean, Lsun, SS, EP

E06A 011. H hour angle 0-360 deg, over Greenwich, = 77.94661, deg = 77, min = 56, sec = 47.80490

E06B 011. H hour angle 0-360 deg, over Lmean, = 348.37753, deg = 348, min = 22, sec = 39.09093

E06C 011. H hour angle 0-360 deg, over Lsun, = 347.94656, deg = 347, min = 56, sec = 47.62898

E06D 011. H hour angle 0-360 deg, over SS, = 0.00000, deg = 0, min = 0, sec = 0.00000

E06E 011. H hour angle 0-360 deg, over EP, = 155.35923, deg = 155, min = 21, sec = 33.21890

07. Finding Delta E is Equation of Time in seconds, at time input UT.

Note - this value in seconds accounts for relative movement of sun in elliptical orbit w.r.t earth and effect of obliquity of the ecliptic;

its maximum value is 16 minutes (960 sec.); Delta E is computed using time in days from the perihelion, n_sun_deg and w_sun at input UT.

Inputs : Time input UT in JD, time perihelion in JD, Sun mean movement n_sun, Eccentricity of earth orbit E_Sun

Outputs : Delta E time_equation in seconds.

E07 011. Delta E Time Equation in seconds = -92.54574, hr = 0, min = 1, sec = 32.54574

08. Finding GST Greenwich sidereal time, and GHA Greenwich hour angle 0 to 360 deg at time when earth is at perihelion.

Inputs : Time in JD when earth at perihelion YY = 2013, MM = 1, DD = 3, hr = 9, min = 11, sec = 56.62

Outputs : GST & GHA in 0-360 deg over Greenwich when earth is at perihelion

E08A 011. GST sidereal time in 0-360 deg over Greenwich at time when earth is at perihelion = 241.14177, hr = 16, min = 4, sec = 34.02452

E08B 011. GHA hour angle in 0-360 deg over Greenwich at time when earth is at perihelion = 241.14717, hr = 16, min = 4, sec = 35.32064

09. Finding ST sidereal time and MST mean sidereal time, over Greenwich, using Earth mean motion rev per day .

Inputs : GST when earth at perihelion, earth rotation rate, ref. JD2000, time input UT in JD, time perihelion in JD.

Outputs : STP, angle perihelion to input JD, ST over Greenwich, MSTO & MST over Greenwich, solar time

E09A 011. STP sidereal time in 0-360 deg over Greenwich when earth at perihelion = 241.14177, hr = 16, min = 4, sec = 34.02452

E09B 011. Angle in 0-360 deg from earth at perihelion to input JD using earth rotational rate = 106.79243,

E09C 011. ST in 0-360 deg over Greenwich using STP and angle from perihelion at input JD = 347.93420, hr = 23, min = 11, sec = 44.20855

E09D 011. ST in 0-360 deg over Greenwich using STP and earth rotation at UT time = 347.93420, hr = 23, min = 11, sec = 44.20855

E09E 011. MSTO in deg, over Greenwich using JD century days, ref J2000 to I/P YY, M1, D1 hr 00 = 100.80714, hr = 6, min = 43, sec = 13.71450

E09F 011. MST in deg, over Greenwich using JD century in days from ref J2000 to UT time Y M D H = 347.94693, hr = 23, min = 11, sec = 47.26242

E09G 011. Solar time over Greenwich in JD (GMT or input UT - 12 hr) = YY 2013, MM 12, DD 21, hr 5, min 10, sec 3.734, ie JD 2456647.71532

10. Finding Earth orbit radius using true anomaly, Sub Sun point (SS) on earth surface and related parameters .

(a) Finding Earth orbit radius using true anomaly.

Inputs : semi-major axis SMA, eccentricity of earth orbit e_sun, sun true anomaly T_Sun

Outputs : earth orbital radius EC to SC (earth center to sun center)

E10A 011. earth orbital radius EC to SC km using true anomaly at UT time = 147162430.9672899200

(b) Finding Sub Sun point (SS) over earth surface (Latitude, Longitude, & Latitude radius) pointing to Sun Ecliptic Log (Lsun), Sun height from earth surface over SS, and LST over SS log at time input UT.

Note - for SS Latitude, used earth inclination, sun true anomaly T_sun and sun argument of perigee w_sun.
for SS Longitude, used Sun right ascension Alpha and sidereal time at Greenwich GST.

Inputs : earth inclination, sun true anomaly T_Sun, sun argument of perigee W_Sun, sun right ascension Alpha, earth equator radius, GST at input UT, log SS & EP, earth orbit radius EC to SC

Outputs : SS point Latitude, Longitude, Latitude radius, LST & LMT over SS .

E10B 011. SS point Latitude +ve or -ve in 0 to 90 deg at UT time = -23.44 ie deg = -23, min = 26, sec = 21.41

E10C 011. SS point Longitude 0 to 360 deg = 282.05 ie deg = 282, min = 3, sec = 12.20

E10D 011. SS point Latitude radius km at UT time = 6374.7700931266

E10E 011. Sun height km from earth surface over SS at UT time = 147156056.1971968100

E10F 011. LST local sidereal time in 0-360 deg over SS log at UT time, (LST = GST + log east) = 270.000 ie hr = 18, min = 0, sec = 0.07540
LST local sidereal time and LMT local mean time with date adjusted to calendar YY MM DD and UT hr mm sec.

E10G 011. LST local sidereal time at Sub Sun point (SS) YY = 2013, MM = 12, DD = 21, hr = 18, min = 0, sec = 0.08

E10H 011. LMT local Mean time at Sub Sun point (SS) YY = 2013, MM = 12, DD = 21, hr = 11, min = 58, sec = 16.55

(c) Finding LST and LMT over Earth point(EP) where Observer is, at time input UT.

Inputs : EP point Latitude, Longitude

Outputs : LST & LMT over EP .

E10I 011. LST local sidereal time in 0-360 deg at EP log at UT time, (LST = GST + log east) = 65.360 ie hr = 4, min = 21, sec = 26.29001
LST and LMT with date adjusted to calendar YY MM DD and UT hr mm sec.

E10J 011. LST local sidereal time at Earth point (EP) YY = 2013, MM = 12, DD = 22, hr = 4, min = 21, sec = 26.29

E10K 011. LMT local Mean time at Earth point (EP) YY = 2013, MM = 12, DD = 21, hr = 22, min = 19, sec = 42.76

Finding Earth to Sun Position Vectors coordinate in PQW, IJK, SEZ frames and the Vector Coordinate Transforms.

First defined coordinate systems, PQW, IJK, SEZ, then computed Position & Velocity vectors in these three coordinate systems.

(a) **Perifocal Coordinate System (POW)**, is Earth Centered Inertial coordinate frame defined in terms of Kepler Orbital Elements.

The system is fixed with time (inertial), pointing towards orbit periapsis;

the system's origin is Earth center (EC), and its fundamental plane is the orbit plane;

the P-vector axis directed from EC toward the periapsis of the elliptical orbit plane,

the Q-vector axis sweeps 90 deg from P axis in the direction of the orbit,

the W-vector axis directed from EC in a direction normal to orbit plane, forms a right-handed coordinate system.

(b) **Geocentric Coordinate System (IJK)**, is also an Earth Centered Inertial (ECI) frame, a Conventional Inertial System (CIS).

The system is fixed with time (inertial), pointing towards vernal equinox;

the system's origin is Earth center (EC), and its fundamental plane is the equator;

the I-vector is +X-axis directed towards the vernal equinox direction on J2000, Jan 1, hr 12.00 noon,

the J-vector is +Y-axis sweeps 90 deg to the east in the equatorial plane,

the K-vector is +Z-axis directed towards the North Pole.

(c) **Topocentric Horizon Coordinate System (SEZ)**, is Non-Inertial coordinate frame, known as Earth-Centered Earth-Fixed Coordinates (ECEF).

The system moves with earth, is not fixed with time (non-inertial), is for use by observers on the surface of earth;

the observer's surface forms the fundamental plane, is tangent to earth's surface

the S-vector is +ve horizontal -axis directed towards South,

the E-vector is +ve horizontal -axis directed towards East,

the Z-vector is +ve normal directed upwards on earth surface.

Note that axis Z not necessarily pass through earth center, so not used to define as radius vector.

11. Finding Earth center(EC) to Sun center(SC) Range Vector[r_p , r_q , r] from in PQW frame, perifocal coordinate system.

Inputs : Semi-major axis (SMA), Eccentricity of earth orbit (e_{sun}), Sun eccentric anomaly (E_{sun})

Outputs : Vector(r , r_p r_q) in PQW frame

E11A 011. r earth pos vector magnitude EC to SC km in PQW frame perifocal cord at UT time = 147162430.96729

E11B 011. rp earth pos vector component EC to SC km in PQW frame perifocal cord at UT time = 143290324.9509236200

E11C 011. rq earth pos vector component EC to SC km in PQW frame perifocal cord at UT time = -33536008.4634748730

Note - r earth pos vector magnitude EC to SC km in PQW frame is same as earth orbital radius computed before using true anomaly.

12. Transform_1 Earth position EC to SC Range Vector[r_p , r_q] in PQW frame To Range Vector[r_I , r_J , r_K] in IJK frame, inertial system cord.

Inputs : Vector(r_p , r_q) EC_to_SC km in frame PQW , Alpha rd, w_{sun} rd, earth_inclination rd ,

Outputs : Vector(r_I , r_J , r_K , r) EC_to_SC km in frame IJK

E12A 011. r_I earth pos vector component EC to SC km frame IJK at UT time = -135018902.5487828300

E12B 011. r_J earth pos vector component EC to SC km frame IJK at UT time = 125.5141403824

E12C 011. r_K earth pos vector component EC to SC km frame IJK at UT time = -58537825.7429337430

E12D 011. r earth pos vector magnitude EC to SC km frame IJK at UT time = 147162430.9672938900

Note - r earth pos vector magnitude EC to SC km in PQW frame is same as that computed above in PQW frame.

13. Transform_2 Earth point EP(lat, log, hgt) To EC to SC Range Vector[R_I , R_J , R_K , R] in IJK frame.

Inputs : earth equator radius_km, earth point EP(lat deg, log deg, hgt meter),

LST_local_sidereal_time_in_0_to_360_deg_at_EP_log_at_UT_time,

Outputs : Vector(R_I , R_J , R_K , R) Range EC to EP in IJK frame

E13A 011. R_I pos vector component EC to EP km frame IJK at UT time = 2441.9770992295

E13B 011. R_J pos vector component EC to EP km frame IJK at UT time = 5323.8052057294

E13C 011. R_K pos vector component EC to EP km frame IJK at UT time = 2517.6312937817

E13D 011. R pos vector magnitude EC to EP km frame IJK at UT time = 6375.3134317570

14. Transform_3 Earth position EC to SC Range Vectors [r_I r_J r_K] & [R_I R_J R_K] To EP to SC Range Vector[rv_I , rv_J , rv_K] in IJK frame.

Inputs : Vector(r_I r_J r_K) position EC_to_SC km in frame IJK , Vector(R_I R_J R_K) range EC to SC km in IJK frame,

Outputs : Vector(rv_I , rv_J , rv_K , rv) range EP to SC in IJK frame

E14A 011. rv_I range vector component EP to SC km frame IJK at UT time = -135021344.5258820700

E14B 011. rv_J range vector component EP to SC km frame IJK at UT time = -5198.2910653470

E14C 011. rv_K range vector component EP to SC km frame IJK at UT time = -58540343.3742275240

E14D 011. rv range vector magnitude EP to SC km frame IJK at UT time = 147165672.9912639600

15. Transform_4 Earth point EP to SC Range Vector[rvI, rvJ, rvK] in IJK frame To EP to SC Range Vector[rvS, rvE, rvZ] in SEZ frame.

Inputs : lat_pos_neg_0_to_90_deg_at_EP_at_time_UT , LST_local_sidereal_time_in_0_to_360_deg_at_EP_log_at_UT_time

Vector(rvI, rvJ, rvK, rv) range EP to SC km in IJK frame,

Outputs : Vector(rvS, rvE, rvZ, rv) range EP to SC km in SEZ frame

E15A 011. rvS range vector component EP to SC km frame SEZ at UT time = 31550012.9630649570

E15B 011. rvE range vector component EP to SC km frame SEZ at UT time = 122724394.7305014100

E15C 011. rvZ range vector component EP to SC km frame SEZ at UT time = -74840195.9312917890

E15D 011. rv range vector magnitude EP to SC km frame SEZ at UT time = 147165672.9912639300

16. Finding Elevation(EL) and Azimuth(AZ) angle of Sun at Earth Observation point EP .

Note : Results **computed using 4 different formulations**, each require different inputs to give EL & AZ angles.

For all situations of Object and Observer positions, a combination of Latitude N/S & Longitude E/W :

Method 1 : for both EL & AZ angles, this does not provide correct results ;

Method 2 : for only EL angle, this provides consistent, unambiguous correct results.

but for AZ angles the results are ambiguous, need corrections by adding or subtracting values as 180 or 360 or sign change.

Method 3 : same as method 2, for EL angle, the results are correct, but for AZ angles the results are ambiguous, need corrections.

Method 4 : for finding Azimuth and Distance but not for finding Elevation angle;

for AZ angles, this provides correct unambiguous results that need no further corrections.

Therefore for Elevation (EL) angle Method 3 results are accepted and for Azimuth (AZ) angle Method 4 results are accepted .

Results verified from other sources; Ref URLs <http://www.ga.gov.au/geodesy/astro/smpos.jsp#intzone> .

NOAA Research <http://www.esrl.noaa.gov/gmd/grad/solcalc/> , and <http://aa.usno.navy.mil/data/docs/AltAz.php>

Xavier Jubier, Member IAU http://xjubier.free.fr/en/site_pages/astronomy/ephemerides.html

Rem: SS point lat deg = -23.44, log deg = 282.05 YY = 2013, MM = 12, DD = 21, hr = 11, min = 58, sec = 16.55

EP point lat deg = 23.26, log deg = 77.41 YY = 2013, MM = 12, DD = 21, hr = 22, min = 19, sec = 42.76

Elevation(EL) & Azimuth(AZ) angle of Sun at Earth Observation point EP, (**method 1** - computed values may be Ambiguous or Incorrect).

Inputs : Vector[rvS, rvE, rvZ] range EP to SC km in SEZ frame

Outputs : Elevation(EL) & Azimuth(AZ) of Sun at EP

E16A 011. Elevation angle deg of Sun at EP using rv SEZ at UT time = -30.5668840082

E16B 011. Azimuth angle deg of sun at EP using rv SEZ at UT time = 255.5825938098

Elevation(EL) & Azimuth(AZ) angle of Sun at Earth Observation point EP, (method 2 - computed AZ values may be ambiguous & incorrect).

Inputs : Time input UT YY MM DD HH, Equator radius, EP lat & log, SS lat & log, Sun declination Delta

Outputs : Elevation(EL) & Azimuth(AZ) of Sun at EP

E16C 011. Elevation angle deg of Sun at EP using Sun declination diff log range EP to SC = -67.40419

E16D 011. Azimuth angle deg of sun at EP using sun declination diff log range EP to SC = -174.61010

Elevation(EL) & Azimuth(AZ) angle of Sun at Earth Observation point EP, (method 3 - computed AZ values may be ambiguous & incorrect).

Inputs : Time input UT YY MM DD HH, Equator radius, EP lat & log, SS lat & log, Sun hgt from EC, Sun range from EP

(Sun hgt from EC = earth orbit radius EC to SC km ; Sun range from EP = rv range vector EP to SC km frame SEZ)

Outputs : Elevation(EL) & Azimuth(AZ) of Sun at EP

E16E 011. Elevation angle deg of Sun at EP using Sun hgt diff log range EP to SC = -67.40528 ie deg = -67, min = 24, sec = 19.03

E16F 011. Azimuth angle deg of sun at EP using sun hgt diff log range EP to SC = 95.39461 ie deg = 95, min = 23, sec = 40.61

Azimuth(AZ) angle of Sun at Earth Observation point EP, (method 4 - computed AZ values is unambiguous & correct).

Inputs : Time input UT YY MM DD HH, EP lat & log, SS lat & log

Outputs : Azimuth(AZ) of Sun at EP

E16G 011. Azimuth angle deg of sun at EP using sun hgt diff log range EP to SC = 264.60539 ie deg = 264, min = 36, sec = 19.39

Due to such incorrect results, finally for Elevation (EL) Method 3 results and for Azimuth (AZ) Method 4 results are accepted.

Finally accepted Elevation angle deg of Sun from EP to SC = -67.4052849205 ie deg = -67, min = 24, sec = 19.03

Finally accepted Azimuth angle_deg of Sun from EP to SC = 264.6053872100 ie deg = 264, min = 36, sec = 19.39

Distance in km from Earth observation point(EP) to Sub Sun point(SS) and Earth Velocity meter per sec in orbit at time input UT.

17. Finding Distance in km from Earth observation point(EP) to Sub Sun point(SS) over Earth surface .

Inputs : EP lat & log, SS lat & log,

Outputs : Distance in km from EP to SS over Earth surface

E17A 011. Distance in km Earth observation point(EP) to Sub Sun point(SS) = 17522.14782

Finding Earth Velocity meter per sec in orbit in frame PQW

Inputs : semi-major axis SMA, GM_Sun, earth pos r EC to SC frame IJK, eccentricity of earth orbit e_Sun, sun eccentric anomaly E_Sun

Outputs : Earth Velocity magnitude and component Xw Yw in frame PQW in meter per sec

E17B 011. Velocity magnitude meter per sec using GM, SMA, r earth EC to SC frame IJK at UT time = 30273.6689861666

E17C 011. Velocity component meter per sec in orbit Xw using GM, e_Sun, SMA, E_Sun at UT time = 6788.3947484369

E17D 011. Velocity component meter per sec in orbit Yw using GM, e_Sun, SMA, E_Sun at UT time = 29502.7580172325

Finding Earth Velocity Vector [vX, vY, vZ] in meter per sec in orbit; a Transform of [Xw, Yw] in frame PQW To [vX, vY, vZ] in frame XYZ

Inputs : velocity component (Xw, Yw), sun right ascension Alpha, Sun Argument of perigee W_Sun, inclination Epcylone

Outputs : earth velocity vector(vX, vY, vZ, vR) meter per sec in frame XYZ

E17E 011. vX earth Velocity vector meter per sec using Xw Yw frame PQW RA w i at UT time = 104.0469608256

E17F 011. vY earth Velocity vector meter per sec using Xw Yw frame PQW RA w i at UT time = -30273.4565603953

E17G 011. vZ earth Velocity vector meter per sec using Xw Yw frame PQW RA w i at UT time = 45.1220543039

E17H 011. vR earth Velocity magnitude meter per sec using Xw Yw frame PQW RA w i at UT time = 30273.6689861666

Earth State Vectors : Position [X, Y, Z] in km and Velocity [Vx, Vy, Vz] in meter per sec, at time input UT.

18. Finding Earth State Position Vector [X, Y, Z] in km at time input UT.

Inputs : position vector(r_I, r_J, r_K, r) in frame IJK values assigned to state position vector

Outputs : State Position Vector(X, Y, Z, R) in km, frame XYZ

E18A 011. State vector position X km at UT time = -135018902.5487828300

E18B 011. State vector position Y km at UT time = 125.5141403824

E18C 011. State vector position Z km at UT time = -58537825.7429337430

E18D 011. State vector position R km at UT time = 147162430.9672938900

19. Finding Earth State Velocity Vector [Vx, Vy, Vz] in meter per sec at time input UT.

Inputs : velocity vector(v_X, v_Y, v_Z, v_R) meter per sec in frame XYZ values assigned to state velocity vector

Outputs : state velocity vector(Vx, Vy, Vz, V) meter per sec, frame XYZ

E19A 011. State vector velocity Vx meter per sec at UT time = 104.0469608256

E19B 011. State vector velocity Vy meter per sec at UT time = -30273.4565603953

E19C 011. State vector velocity Vz meter per sec at UT time = 45.1220543039

019D 011. State vector velocity V meter per sec at UT time = 30273.6689861666

20. Earth Orbit Normal Vector [Wx, Wy, Wz] in km and angles Delta, i, RA at time input UT; Normal is line perpendicular to orbit plane.

Inputs : earth pos r EC to SC frame IJK, inclination Epcylone, sun right ascension Alpha

Outputs : earth orbit normal vector (Wx, Wy, Wz, W) in km

E20A 011. Earth orbit normal W km using r earth pos frame IJK inclination Alpha = 147162430.9672938900

E20B 011. Earth orbit normal Wx km using r earth pos frame IJK inclination Alpha = -58537825.7429084480

E20C 011. Earth orbit normal Wy km using r earth pos frame IJK inclination Alpha = 54.4170093468

E20D 011. Earth orbit normal Wz km using r earth pos frame IJK inclination Alpha = 135018902.5488411500

020E 011. Earth orbit normal Delta W deg using r earth pos frame IJK inclination Alpha = 66.5607205617

E20F 011. Earth orbit normal Inclination i deg using normal_Delta_W = 23.4392794383

E20G 011. Earth orbit normal Alpha W deg using r earth pos frame IJK, inclination, Alpha = -0.0000532624

E20H 011. Earth orbit normal Right ascension of ascending node using normal Alpha, W = 89.9999467376

Transform Earth State Vectors to Earth position Keplerian elements.

21. Finding Earth position Keplerian elements computed using State Vector, at time input UT.

Inputs : State vector year, days decimal of year, revolution, node, State Position Vector [X, Y, Z], State Velocity Vector [Vx, Vy, Vz]

Outputs : Keplerian elements : year, days decimal of year, revolution, node, inclination, right ascension, eccentricity, argument of perigee, mean anomaly, mean motion rev per day, mean angular velocity rev per day, mean motion rev_per_day from SMA considering oblateness

E21A 011. Keplerian elements year = 2013, days_decimal_of_year = 354.71532, revolution no = 1, node = 2 ie decending

E21B 011. inclination_deg = 23.4392794383

E21C 011. right ascension ascending node deg = 269.9999467376

E21D 011. eccentricity = 0.0167102190

E21E 011. argument of perigee_deg = 283.1725439084

E21F 011. mean anomaly deg = 347.2585513649

E21G 011. mean_motion rev per day = 0.0027377786

E21H 011. mean angular velocity rev_per_day = 0.0027377786

E21I 011. mean motion rev per day using SMA considering oblateness = 0.0027377786

Transform Earth position Keplerian elements to Earth State Vectors .

22. Finding Earth position State Vectors, computed using Keplerian elements at time input UT

(computed again to validate model equations, Keplerian elements to State Vectors & back)

Inputs : Keplerian elements : year, days decimal of year, revolution, node, inclination, right ascension, eccentricity, argument of perigee, mean anomaly, mean motion rev per day, mean angular velocity rev per day, mean motion rev_per_day from SMA considering oblateness

Outputs : State vector year, days decimal of year, revolution, node, State Position Vector [X, Y, Z], State Velocity Vector [Vx, Vy, Vz]

E22A 011. State vectors year = 2013, days_decimal_of_year = 354.71532, revolution no = 1, node = 2 ie decending

E22B 011. state vector position X km = -135018902.5487863400, state vector velocity Vx meter per sec = 104.0469606286

E22C 011. state vector position Y km = 125.5152014755, state vector velocity Vy meter per sec = -30273.4565603953

E22D 011. state vector position Z km = -58537825.7429352700, state vector velocity Vz meter per sec = 45.1220542184

E22E 011. state vector position R km = 147162430.9672977600, state vector velocity V meter per sec = 30273.6689861658

Note : Computation of all above parameters, grouped in 1 to 22, corresponds to time

- (a) Universal time over Greenwich (UT/GMT) : Year = 2013, Month = 12, Day = 21, Hour = 17, Min = 10, Sec = 3.734
- (b) Mean Solar time (MST) over Earth Observation point (EP) : Year = 2013, Month = 12, Day = 21, Hour = 22, Min = 19, Sec = 42.762

Thus Computed values for Position of Earth on Celestial Sphere corresponding to Standard Epoch time JD2000, and at six Astronomical Events while Earth reaches Perihelion, Vernal equinox, Summer solstice, Aphelion, Autumnal equinox, Winter solstices.

A Summary of these Computed values are presented next.

Next Section - 4.8 Concluding position of earth at six astronomical events

Concluding Position of Earth on Celestial Sphere (Sections 4.0 to 4.7).

Summary of Earth Position on Celestial Sphere with respect to Sun, at Std Epoch J2000 and at six astronomical events in the Year = 2013.

In previous Sections (4.1 to 4.7), presented Earth Position on Celestial Sphere with respect to Sun, at Std Epoch J2000 and in year 2013 when Earth was at Perihelion, Vernal equinox, Summer solstice, Aphelion, Autumnal equinox, Winter solstices.

The Earth Observation point (EP) considered as : Latitude deg = 23.25993, Longitude deg = 77.41261, height km = 0.49470 .

The Summary of Computed values presented below, are put in three parts as :

- UT/GMT, LMT at Lat/Log of SS, Sun EL & AZ angles at EP, Surface distance EP to SS, Earth Vel;
- Earth position Keplerian elements : Inclination, RA of asc. node, Eccentricity, Arg. of Perigee, Mean Anomaly, Mean Motion ;
- Earth State Vectors : Position (X, Y, Z) & Velocity (Vx, Vy, Vz) .

(a) UT/GMT, LMT at Lat/Log of SS, Sun EL & AZ angles at EP, Surface distance EP to SS and Earth Vel, values at different orbit points

Earth Orbit Events	Universal time (GMT Dt/time)	SubSun Point (SS) (LMT Dt/time Lat/Log)	Sun hgt at SS (km)	Earth Point (EP) (LMT Dt/time)	Sun angle at EP (EL/AZ deg)	Dist. EP to SS (km)	Earth Vel. (m/s)
Epoch J2000	2000.01.01/ 12:00:00	2000.01.01/ 12:03:18 -23.034 0.826	147094821.78	2000.01.01/ 17:09:39	2.382 243.631	9753.28	30286.06663
Perihelion	2013.01.03/ 09:11:56	2013.01.03/ 12:04:32 -22.789 43.150	147092415.73	2013.01.03/ 14:21:35	33.164 218.319	6326.74	30286.55386
Vernal equinox	2013.03.20/ 11:02:09	2013.03.20/ 12:07:26 0.000 16.321	148983652.48	2013.03.20/ 16:11:48	26.365 257.698	7083.54	29906.03259
Summer Solstice	2013.06.21/ 05:01:19	2013.06.21/ 12:01:45 23.439 105.111	152024178.61	2013.06.21/ 10:10:58	64.608 275.977	2826.53	29304.29892
Aphelion	2013.07.05/ 00:18:52	2013.07.05/ 12:04:32 22.788 176.418	152092067.01	2013.07.05/ 05:28:31	1.165 245.612	9888.77	29290.99989
Autumnal equinox	2013.09.22/ 20:45:38	2013.09.22/ 11:52:31 -0.000 226.719	150121946.39	2013.09.23/ 01:55:17	-52.189 236.365	15828.21	29679.34041
Winter solstice	2013.12.21/ 17:10:03	2013.12.21/ 11:58:16 -23.439 282.053	147156056.20	2013.12.21/ 22:19:42	-67.405 95.395	17522.15	30273.66899

(b) Earth Position Keplerian Elements , values at different orbit points

Earth Orbit Events	Epoch (year, day of year)	Inclination (deg)	RA of asc. node (deg)	Eccentricity (deg)	Arg. of Perigee (deg)	Mean Anomaly (deg)	Mean Motion (deg)
Epoch J2000	2000 0.50000	23.43928	281.28586	0.0167102190	282.93203	357.52800	0.0027377786
Perihelion	2013 2.38329	23.43928	284.29222	0.0167102190	283.15577	0.00000	0.0027377786
Vernal equinox	2013 78.45983	23.43928	0.00000	0.0167102190	283.15977	74.98105	0.0027377786
Summer Solstice	2013 171.20925	23.43928	89.99994	0.0167102190	283.16363	166.39491	0.0027377786
Aphelion	2013 185.01311	23.43928	104.30150	0.0167102190	256.83563	180.00000	0.0027377786
Autumnal equinox	2013 264.86503	23.43928	180.00000	0.0167102190	103.16768	258.70208	0.0027377786
Winter solstice	2013 354.71532	23.43928	269.99995	0.0167102190	283.17254	347.25855	0.0027377786

(c) Earth State Vectors - position & velocity , values at different orbit points

Earth Orbit Events	Position X vector (km)	Position Y vector (km)	Position Z vector (km)	Position R mag. (km)	Velocity Vx vector (m/s)	Velocity Vy vector (m/s)	Velocity Vz vector (m/s)	Velocity V mag. (m/s)
Epoch J2000	-125003885.45	-51961735.98	-57556656.24	147101196.66	10756.75	-28227.52	2178.39	30286.07
Perihelion	-119085952.63	-64886279.60	-56976844.62	147098790.67	13409.17	-27017.61	2741.95	30286.55
Vernal equinox	148990030.62	0.00	0.00	148990030.62	484.70	27434.65	11894.37	29906.03
Summer Solstice	-139485317.94	137.15	60474252.72	152030553.38	-104.03	-29304.08	45.09	29304.30
Aphelion	-140226215.40	446.26	58911327.94	152098441.95	1114.70	-29149.25	2653.53	29291.00
Autumnal equinox	-150128324.53	-0.00	-0.00	150128324.53	484.69	-27226.63	11804.18	29679.34
Winter solstice	-135018902.55	125.52	-58537825.74	147162430.97	104.05	-30273.46	45.12	30273.67

The computed values presented in table (a) (b) (c) show consistency. The angles are expressed in deg and distances in km.

End of computing position of Earth on Celestial Sphere at input UT Standard Epoch time JD2000 and at six astronomical events.

Next Section - 5 Satellites Orbit around Earth : Ephemeris data set.

REFERENCES : TEXT BOOKS & INTERNET WEB LINKS.

Books

1. Dennis Roddy, 'Satellite Communication', Third Edition, McGraw Hill, chap. 2 - 3, pp 21-86, Jan 2001.
2. Gerard Maral, Michel Bousquet, 'Satellite Communications Systems', Fifth Edition, John Wiley & Sons, chap. 2, pp 19-97, 2002.
3. Hannu Karttunen, Pekka Kroger, et al, 'Fundamental Astronomy', Springer, 5th Edition, pp 1 - 491, 2007.
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3. Keith Burnett, 'Converting RA and DEC to ALT and AZ', Approximate Astronomical Positions, Last Modified May 27, 1998, URL <http://www.stargazing.net/kepler/al taz.html>
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6. Hartmut Frommert, 'General Coordinate Systems', Index of /spider/ScholarX, spider.seds.org/spider, URL http://spider.seds.org/spider/ScholarX/coord_bas.html

ANNEXURE : A Collection of few OM-MSS related Diagrams / Help.

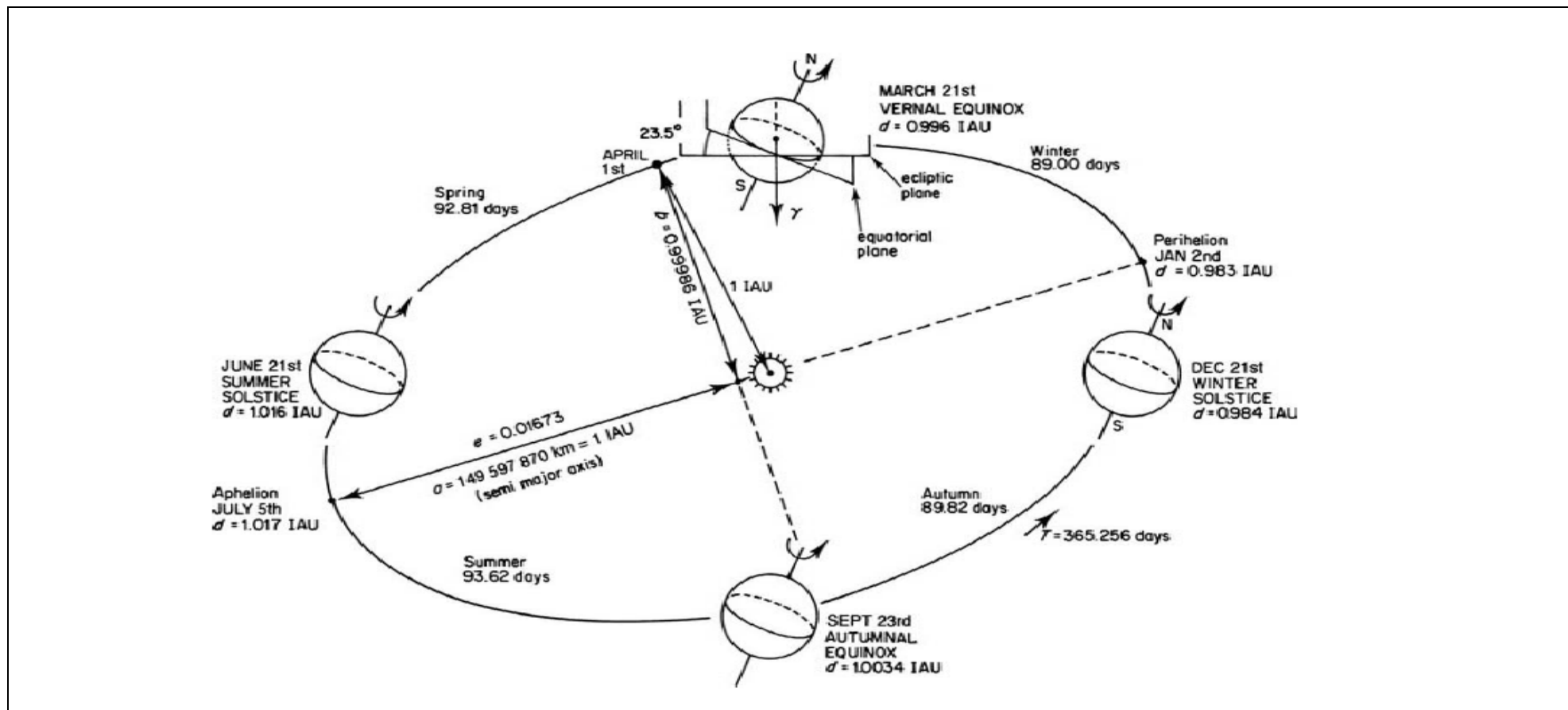


Fig-3. Orbit of Earth Around Sun

Earth rotates around sun with a period of approximately 365.25 days following an Ellipse of Eccentricity 0.01673 and Semi -major axis 149597870 km, which defines the Astronomical unit of distance (AU). Around 2 January, Earth is nearest from sun called **Perihelion** while around 5 July it is farthest from Sun called **Aphelion** (around 152100000 km). The other events point are **Vernal equinox** around 21 March, **Autumnal equinox** around 23 September, **Summer solstice** around 21 June and **Winter solstic** around 21 December. The plane of the orbit is called the plane of the Ecliptic that makes an angle 23.44 deg (the Obliquity of the Ecliptic) with the mean Equatorial plane.

Source Book by Gerard Maral, Michel Bousquet, 'Satellite Communications Systems', Fifth Edition, John Wiley & Sons, chap. 2, Pg 29, 2002.

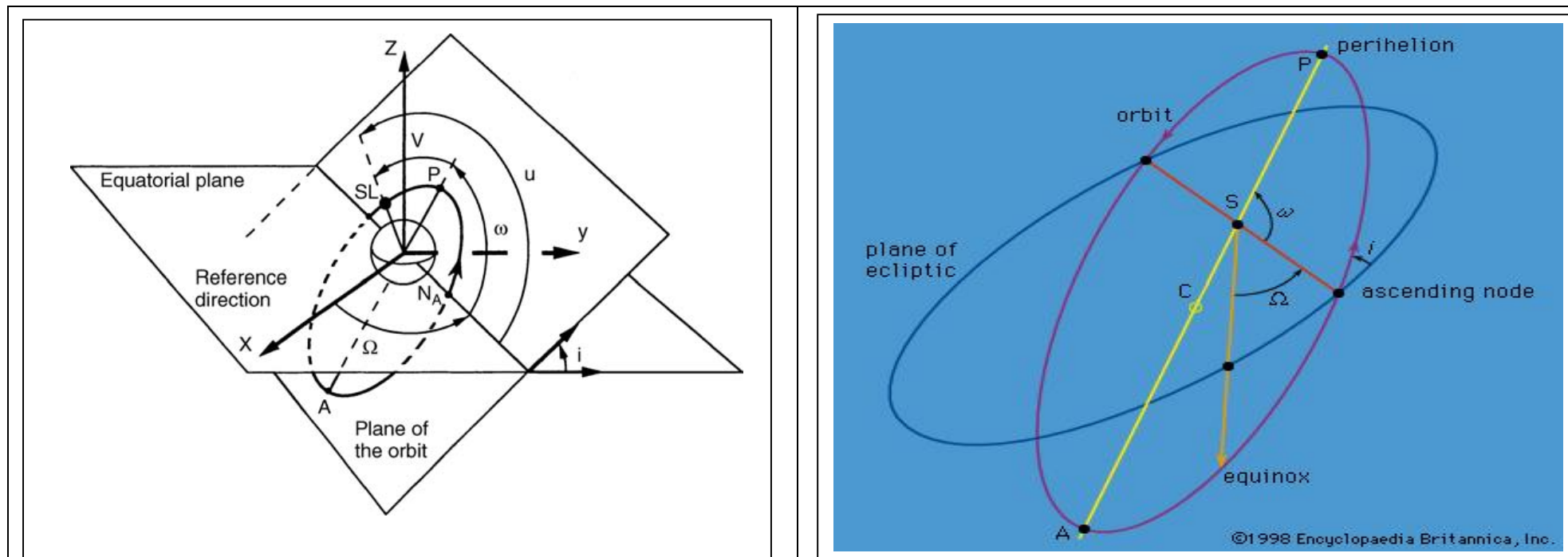


Fig- 4 & 5 Positioning of Orbit in Space

Orbit Position in Space at Epoch is defined by the Values of Kepler Orbit elements : (definitions apply to both planets & Satellites)

1. **Inclination 'i'** of the orbit of a planet, is angle between the plane of planet's orbit and the plane containing Earth's orbital path (ecliptic).
2. **Right ascension 'Ω'** of the ascending node is the angle taken positively from 0 to 360 deg in the forward direction, between the reference direction and the ascending node of the orbit (the intersection of the orbit with the plane of the equator crossing this plane from south to north).
3. **Argument of Perigee 'ω'**, specify angle between orbit's perigee and orbit's ascending node, measured in orbital plane and direction of motion.
4. **Eccentricity 'e'** of an orbit shows how much the shape of an object's orbit is different from a circle;
5. **Mean Anomaly 'v'** relates the position and time for a body moving in a Kepler orbit. The mean anomaly of an orbiting body is the angle through which the body would have traveled about the center of the orbit's auxiliary circle. 'M' grows linearly with time.

A knowledge of above five parameters completely defines the trajectory of an object or satellite in space. However, the **Nodal angular elongation 'u'** can also be used to define the position of the satellite in its orbit. This is the angle taken positively in the direction of motion from 0 to 360 deg between the direction of the ascending node and the direction of the satellite ($u = \omega + v$).

Source Book by Gerard Maral, Michel Bousquet, 'Satellite Communications Systems', Fifth Edition, John Wiley & Sons, chap. 2, Pg 29, 2002. & <http://www.britannica.com/EBchecked/topic/101285/celestial-mechanics/images-videos/2285/orbital-element-keplers-laws-of-planetary-motion>