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POSITIONAL ASTRONOMY : EARTH ORBIT AROUND SUN

RC Chakraborty (Retd), Former Director, DRDO, Delhi & Visiting Professor, JUET, Guna,

www.myreaders.info, rcchak@gmail.com,

www.myreaders.info/html/orbital_mechanics.html, Revised Dec. 16, 2015

(This is Sec. 2, pp 33 - 56, of Orbital Mechanics - Model & Simulation Software (OM-MSS), Sec 1 to 10, pp 1 - 402.)

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POSITIONAL ASTRONOMY : EARTH ORBIT AROUND SUN, ASTRONOMICAL EVENTS ANOMALIES, EQUINOXES, SOLSTICES, YEARS & SEASONS.

Look at the Preliminaries about 'Positional Astronomy', before moving to the predictions of astronomical events.

Definition : Positional Astronomy is measurement of Position and Motion of objects on celestial sphere seen at a particular time and location on Earth.

Positional Astronomy, also called Spherical Astronomy, is a System of Coordinates.

The Earth is our base from which we look into space. Earth orbits around Sun, counterclockwise, in an elliptical orbit once in every 365.26 days.

Earth also spins in a counterclockwise direction on its axis once every day. This accounts for Sun, rise in East and set in West.

Term 'Earth Rotation' refers to the spinning of planet earth on its axis. Term 'Earth Revolution' refers to orbital motion of the Earth around the Sun.

Earth axis is tilted about 23.45 deg, with respect to the plane of its orbit, gives four seasons as Spring, Summer, Autumn and Winter.

Moon and artificial Satellites also orbits around Earth, counterclockwise, in the same way as earth orbits around sun.

Earth's Coordinate System : One way to describe a location on earth is Latitude & Longitude, which is fixed on the earth's surface.

The Latitudes and Longitudes are presented in several ways. Example, location of Delhi, India,

Using degree decimal	latitude 28.61	North of Equator,	longitude 77.23	East of Greenwich
Using degree minutes second	latitude 28:36:36	North of Equator,	longitude 77:13:48	East of Greenwich
Using time zone hour minutes second	latitude 28:36:36	North of Equator,	longitude 05 hours, 8 min, 55.2 sec	East of Greenwich

(east of Greenwich, means in Delhi sun will set at 05 hours, 8 min, 55.2 sec before it sets in Greenwich, ie at Delhi UTM is +05 hours, 8 min, 55.2 sec)

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Laws of Planetary Motion : In the early 1600s, Johannes Kepler proposed three laws of planetary motion.

1st The Law of Ellipses : The orbits of the planets are ellipses, with the Sun at one focus of the ellipse.

2nd The Law of Equal Areas : The line joining the planet to the Sun sweeps out equal areas in equal times as the planet travels around the ellipse.

3rd The Law of Harmonies : The ratio of squares of revolutionary periods for two planets is equal to ratio of the cubes of their semi-major axes.

Kepler's first law says all planets orbit the sun in a path that resembles an ellipse.

Kepler's second law describes the speed at which any given planet will move while orbiting the sun; this speed is constantly changing.

A planet moves fastest when it is closest to the sun, and a planet moves slowest when it is furthest from the sun.

Kepler's third law compares motion characteristics of different planets; for every planet, the ratio of squares of their periods to the cubes of their average distances from the sun is the same. It implies that the period for a planet to orbit the Sun increases rapidly with the radius of its orbit. Thus, Mercury the innermost planet, takes only 88 days to orbit the Sun but the outermost planet (Pluto) requires 248 years to do the same.

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Glossary of terms : definitions, meaning and descriptions

1. Celestial Sphere, is an imaginary rotating sphere of infinite radius, concentric with Earth.

   The poles and equator of the celestial sphere are the projections of earth's poles and equator out into space, called Celestial North Pole, Celestial South Pole and Celestial Equator.

2. Celestial coordinate, is a system for specifying positions of celestial objects, the satellites, planets, stars, galaxies, and more.

   The common celestial coordinate systems are : Horizontal, Equatorial, and Ecliptic; two others, Galactic and Supergalactic not included here.

   Each coordinate system is named for its choice of fundamental plane as reference plane.

   - Horizontal coordinate system uses the observer's horizon as the fundamental reference plane.

     The coordinates of a point on celestial sphere are Altitude or Elevation and Azimuth.

     Altitude (Alt) also referred as Elevation (EL) is the angle between the object and the observer's local horizon, expressed as 0 to 90 deg, +ve/-ve.

     Azimuth (Az) is the angle of the object around the horizon, usually measured from the north increasing towards the east.

   - Equatorial coordinate system uses the celestial equator as primary reference circle.

     The coordinates of a point on celestial sphere are Declination and Right ascension, analogous to Latitude-Longitude coordinate system used on Earth.

     Declination (delta) of an object is angle measured from celestial equator (0 deg declination) along a meridian line through the object.

     Right ascension (RA) lines on celestial sphere are identical to longitude lines (0-360 deg) on Earth, but the differences are that

     (i) the RA lines on the celestial sphere divide one rotation into 24 hours (one hour = 15 deg) expressed in terms of hours : minutes : seconds.

     (ii) for RA the start point for zero hour is vernal equinox or first point of Aries where Sun crosses celestial equator and not 0 deg log.

   - Ecliptic coordinate system uses the ecliptic, as the fundamental reference plane.

     The ecliptic is apparent path of Sun on celestial sphere, crosses celestial equator twice in a year, at Autumnal and Vernal equinoxes.

     Coordinates of a point on celestial sphere are Ecliptic latitude and Ecliptic longitude; distance is also necessary for complete spherical position.

     Ecliptic latitude (Lat) is the angle between a position and ecliptic, takes values between -90 and +90 deg.

     Ecliptic longitude (Log) starts from the vernal equinox or first point of Aries as 0 deg and runs to 360 de

   These three celestial coordinate systems are summarized below.

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The Summary of Celestial Coordinate Systems :

Systems	Coordinates (Vertical	Horizontal)	Center point (Origin)	Ref. Plane (0 deg Vertical)	Poles	Ref. Direction (0 deg horizontal)
(a) Horizontal	Altitude or elevation,	Azimuth	Observer on earth	Horizon	Zenith/nadir	North or south point of horizon
(b) Equatorial	Declination	Right ascension or hour angle	Earth center(geocentric)/ Sun center(heliocentric)	Celestial equator	Celestial poles	Vernal equinox
(C) Ecliptic	Ecliptic latitude	Ecliptic longitude	Earth center(geocentric)/ Sun center(heliocentric)	Ecliptic	Ecliptic poles	Vernal equinox

3. Celestial Orbit in astronomy, is a gravitationally curved path of celestial body around a point in space.

The orbits of planets around the sun or the orbits of satellites around planets are typically elliptical, governed by Kepler's laws of motion.

The orbit of each planet is influenced by the other planets as well as by the sun, to a small degree, called perturbations.

These perturbations are taken into account in calculating planetary orbits.

- Heliocentric orbit is an orbit around the Sun. The planets, comets, and asteroids in our Solar System are in such orbits.

- Geocentric orbit is an orbit around the Earth. The Moon and all artificial satellites are in such orbits.

- Periapsis & Apoapsis, Perihelion & Aphelion, Perigee & Apogee, represent two points on orbit, named differently to identify the body being orbited.

  The point closest to the orbited body is called the periapsis and the point furthest to the orbited body is called the Apoapsis.

  Perihelion and Aphelion, refer to orbits around the Sun; here the orbit point closest to Sun is perihelion and point furthest to Sun is aphelion.

  Perigee and Apogee, refer to orbits around the Earth; here the orbit point closest to Earth is perigee and point furthest to earth is apogee.

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4. Orbit Elements or Parameters, uniquely identify a specific orbit. There are different ways to describe mathematically the same orbit.

The orbital parameters are usually expressed either by Keplerian elements or by State vectors, each consisting a set of six parameters.

The State vectors, also called Cartesian coordinates, are time-independent, represent the 3-D Position and Velocity components of the orbital trajectory.

Keplerian elements are valid only for a specific time, describe the size, shape, and orientation of an orbital ellipse.

The State Vectors are often not a convenient way to represent an orbit, hence Keplerian elements are commonly used instead.

However, State Vectors and Keplerian Elements can be computed from one another.

(a) State vectors are, three Positions (x, y, z) and three Velocities (x dot, y dot, z dot) at Epoch time t.

- Position vector describe position of the orbiting body in inertial frame of reference, x-axis pointing to vernal equinox and z-axis pointing upwards.
- Velocity vector is velocity of the orbiting body derived from orbital position vector by differentiation with respect to time.

(b) Kepler elements are, Inclination, Longitude of ascending node, Argument of periapsis, Eccentricity, Semimajor axis, Mean Anomaly at Epoch time t.

- Inclination 'i' of the orbit of a planet, is angle between the plane of planet's orbit and the plane containing Earth's orbital path (ecliptic) or with respect to another plane such as the Sun's equator. For Earth-bound observers the ecliptic is more practical;

  e.g. inclination of earth orbit to elliptic is 0 deg and to Sun's equator is 7.155 deg.

- Longitude of the Ascending node, specify orbit of an object in space. For a geocentric orbit, this longitude is called Right Ascension (RA).

  It is the angle from a reference direction, called the origin of longitude, to the direction of the ascending node, measured in a reference plane.

  The reference plane for a Geocentric orbit is Earth's equatorial plane, and the First Point of Aries is the origin of longitude.

  The reference plane for a Heliocentric orbit is Ecliptic plane, and the First Point of Aries is the origin of longitude.

  The angle is measured counterclockwise from the origin to the object.

- Argument of periapsis, specify angle between orbit's periapsis and orbit's ascending node, measured in orbital plane and direction of motion.

  Angle 0 deg, means orbiting body is at its closest to central body, at that moment it crosses the plane of reference from south to north.

  Angle 90 deg, means the orbiting body will reach periapsis at its northmost distance from the plane of reference.

  Adding the argument of periapsis to the longitude of the ascending node gives the longitude of the periapsis.

  The word periapsis is replaced by perihelion (for Sun-centered orbits), or by perigee (for Earth-centered orbits).

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- Eccentricity 'e' of an orbit shows how much the shape of an object's orbit is different from a circle; eccentricity 'e' vary between 0 and 1.

  For Circular orbit : e = 0, elliptical orbit : 0 < e < 1, parabolic trajectory : e = 1, hyperbolic trajectory : e > 1 .

  The Earth's orbital eccentricity varies, from a min value = 0.005 (near circular) to maxi value e = 0.057 (quite elliptical), over a period of 92,000 years, due to gravitational force exerted by Jupiter. The eccentricity of the Earth's orbit is currently about 0.016710219.

- Semimajor axis is one half of the major axis, is the radius of an orbit at the orbit's two most distant points.

  The semi-major axis length 'a' of an ellipse is related to the semi-minor axis length 'b' through the eccentricity 'e'.

- Mean Anomaly 'M' relates the position and time for a body moving in a Kepler orbit. The mean anomaly of an orbiting body is the angle through which the body would have traveled about the center of the orbit's auxiliary circle. 'M' grows linearly with time.

  'M' is a product of orbiting body's mean motion and time past perihelion, where mean motion 'n' = (2. pi / duration of full orbit).

- Epoch is a moment in time, a reference point, for time-varying astronomical quantity, like celestial coordinates or elliptical orbital elements.

5. Heliocentric Orbit : An orbit around the Sun; all planets, comets, asteroids in our solar system are in such orbits.

   Consider the Orbit of Earth around Sun. The one orbit revolution (360 deg), is one sidereal year, occurs every 365.256363 mean solar days, where one solar day = 24h 00m 00s = 24 x 60 x 60 = 86400.00 seconds. The Earth's Orbit Characteristics and Events are :

   (a) Orbit Characteristics : Epoch at J2000.0, **Ref. http://cdn.preterhuman.net/texts/thought_and_writing/reference/wikipedia_2006_CD/wp/e/Earth.htm**

- Aphelion	152,097,701 km	- Perihelion	147,098,074 km
- Semi-major axis	149,597,887.5 km	- Semi-minor axis	149,576,999.826 km
- Axial tilt	23.4392794383 deg	- Eccentricity	0.016710219
- Inclination	7.25 deg to Sun's equator	- Longitude of ascending node	348.73936 deg
- Argument of periapsis	114.20783 deg	- Sidereal orbit period	365.256363 days
- Orbital circumference	924,375,700 km	- average speed	29.78 km/sec

Note : Inclination angle, Longitude of ascending node and Argument of perigee describe the orientation of an orbit in space.

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(b) Orbit Events : Equinox, Solstice, and Seasons

- Equinoxes occur twice a year; Vernal equinox is around 20-21 March and Autumnal equinox is around 22-23 September;

  When equinox occurs, the plane of Earth's equator passes the centre of Sun, i.e. when subsolar point is on Equator;

  At equinox time, the tilt of the Earth's axis is inclined neither away from nor towards the Sun, resulting day and night of same length.

  At equinoxs the Sun is at one of two opposite points on the celestial sphere where the celestial equator (decl 0 deg) and ecliptic intersect;

  One intersection is called vernal point (RA = 00h 00m 00s and log = 0 deg) and other called autumnal point (RA = 12h 00m 00s and log = 180 deg).

- Solstices occur twice a year; Summer solstice is around 21-22 June and Winter solstic is around 21-22 December;

  On solstices day, Sun appears to have reached its highest or lowest annual altitude in the sky above the horizon at local solar noon;

  The solstices day is either longest of the year in summer or the shortest of the year in winter for any place outside of the tropics.

- Seasons occur because the Earth's axis of rotation is not perpendicular to its orbital plane but makes an angle of about 23.439 deg;

  The four Seasons, Spring, Summer, Autumn, Winter, are subdivision of a year, connected with the solstices, and equinoxes;

  The solstices and equinoxs are the four changing points, in the Solar Cycle, that mark the mid-point of the seasons change.

6. Mean anomaly (M), Eccentric anomaly (E), True anomaly (V)

In astronomy, the term anomaly means irregularity in the motion of a planet by which it deviates from its predicted position.

Therefore, astronomers use term anomaly (instead of angle) when calculating the position of objects in their orbits.

Kepler distinguished three kinds of anomaly - mean, eccentric, and true anomaly.

(a) True anomaly is observed angle, as seen from the Sun, between the Earth and the perihelion of the Earth orbit.

When the True anomaly is equal to 0 degrees, then the Earth is closest to the Sun (ie at Perihelion).

When the True anomaly is equal to 180 degrees, then the Earth is furthest from the Sun (ie at Aphelion).

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(b) Mean anomaly is calculated angle, what the true anomaly would be if the Earth moved with constant speed along a perfect circular orbit around the Sun in the same time. Like true anomaly, the mean anomaly is equal to 0 in the perihelion and to 180 degrees in the aphelion, but at other points along the Earth's orbit the true and mean anomalies are not equal to one another. The mean anomaly is often used for one of the orbital elements.

(c) Eccentric anomaly is an auxiliary angle, used to solve Kepler's Equation to find True anomaly from Mean anomaly. The is related to both the Mean and the True anomaly.

Observe equations & relations among Mean anomaly(M), Eccentric anomaly(E), True anomaly(V), Eccentricity(e), Radial disance(r), Semi-major axis(a)

- equation M = E-e.sin(E); a relation between Eccentric and Mean anomaly, is Kepler's equation, solved by numerical methods, e.g. Newton-Raphson method;

- equation cos(v) = {cos(E)-e}/{1-e.cos(E)} ; a relation between Eccentric anomaly and True anomaly ;

- equation r = a {(1-(e sqr)}/{1+e.cos(V)} ; a relation between True anomaly and Radius distance from focus of attraction to the orbiting body.

Thus explained few preliminaries about positional astronomy.

Move on to Motion Of Earth Around Sun - Prediction of Astronomical Events, Anomalies, Equinoxes, Solstices, Years & Seasons.

The precise time of occurrence of following astronomical events are presented in Sections (2.1 to 2.11) respectively :

(a) Earth orbit Mean anomaly, Eccentric anomaly, True anomaly;

(b) Earth reaching orbit points, Perihelion, Aphelion, Vernal Equinox, Autumnal Equinox, Summer Solstice, Winter Solstice;

(c) Earth reaching orbit points, Semi-Major Axis, Semi-Minor Axis;

(d) Astronomical years, Anomalistic, Tropical, Sidereal Years;

(e) Earth orbit oblateness, Semi-Major Axis, Semi-Minor Axis;

(f) Four Seasons, start time of Spring, Summer, Autumn, Winter.

Next Section - 2.1 Earth Orbit Constants used in computation

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Earth Orbit : Constants used in OM-MSS Software.

Astronomical and other Constants used in OM-MSS Software .

Recall the Preliminaries about 'Positional Astronomy', depictions and interpretations, mentioned in section 2 .

(a) The International System of Units (SI), the world's most widely used modern metric system, is followed in OM-MSS Software.

Greenwich meam time (GMT), is identified as Universal time (UT) based on sidereal time at Greenwich, with day starting at midnight.

Standard Epoch J2000, is Julian Day 2451545.00 UT, ie Year 2000, MM 1, hr 12, min 0, sec 0.0 expressed in UT, is Reference point for Time.

One solar day is 24h 00m 00s, ie 24x60x60 = 86400 SI is time for slightly more than one earth rotation, ie 360.9856473356 deg.

One sidereal day is 23h 56m 4.090538155680s = 86164.090538155680s is time for one earth rotation, by exact 360 deg.

One solar year is time period of earth around sun, that vary slightly year to year. Solar year is called astronomical or tropical or civil year.

Consecutive 400 civil years have 97 leap years, so one civil year is (400 x 365 + 97)/400 = 365.2425 mean solar days where

Mean solar day is division of time equal to 24 hours representing average length of the period during which earth makes one rotation on its axis.

One sidereal year (365.256363004 days) is slightly longer than a mean solar year (365.2425 days).

One sidereal year corresponds to 365 day, 6 hr, 9 min, 9.7635456 sec of mean solar time.

One Julian year is exactly 365.25 SI days where SI day is 86400 SI seconds, thus Julian year = 31557600 SI seconds.

Gregorian year is the mean duration of a year of our calendar is 365.2425 SI days, is 31556952 SI seconds.

Earth mean motion rev per day around sun = 1.0 / One sidereal year in days = 1.0 / 365.256363004.

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(b) The Other Constants used in OM-MSS software : the values assigned, correspond to Standard Epoch Julian day JD2000, unless otherwise specified.

RADIAN 57.29577951308232300	Pi 3.141592653589793100
GM_SUN (Gravitational parameter) 132712440018 km3/sec2	GM_EARTH (Gravitational parameter) 398600.4418 km3/sec2
EARTH_EQ_RAD_KM (Earth equator radious) 6378.144 km	EARTH_AVR_RAD_KM (Earth average radious) 6371.0 km
SUN_GEOCENTRIC_DIST_km_from_earth_center 149676781.6 km	MOON_GEOCENTRIC_DIST_km_from_earth_center 381191.7836 km
EARTH_INCLINATION_deg 23.4392794383 deg	EARTH_ORBIT_ECCENTRICITY 0.016710219
EARTH_solar_year 365.2425 days	EARTH_SIDEREAL_YEAR 365.256363004 days
EARTH mean solar day 86400 sec is 24h 00m 00s	EARTH mean sidereal day 86164.0905381557 sec is 23h 56m 4.0905381557
EARTH_MEAN_MOTION_rev_per_day 0.0027378030 rev per day	EARTH_ROTATIONAL_RATE_rad_per_sec 7.2921151467e-5 rad/sec

Move on to Earth Orbit around Sun, : Compute Anomalies, Precise time at Astronomical Events, Years & Seasons.

Next Section - 2.2 Earth orbit - mean and true anomaly

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Earth Mean anomaly and True anomaly at Input UT, Since Standard Epoch J2000, using standard analytical expressions.

Finding Mean anomaly and True anomaly in deg, since Standard Epoch J2000 = 2451545.0 Julian day.

Mean anomaly gives the planet's angular position for a circular orbit with radius equal to its Semi Major Axis.

Since Earth orbit is elliptic, the speed of the Earth varies and the mean anomaly is inaccurate with about +/- 2 deg.

The mean anomaly M = 357.5291 + (0.98560028 * (ip_julian_day - 2451545.0)), is measured from 0 to 360 deg.

During one revolution the mean anomaly values are : at perigee mean anomaly = 0 deg , and at apogee mean anomaly = 180 deg .

True anomaly is true positon of the earth relative to its perihelion, is measured from 0 to 360 deg.

Note that what is true anomaly, would be Mean anomaly if the Earth moved with constant speed along a perfectly circular orbit around Sun.

Equation of Center, is the difference between the true anomaly and the mean anomaly.

Standard Epoch J2000, is beginning of Year = 2000, Month = 1, Day of month = 1, Hour decimal = 12.0 .

Finding Mean anomaly and True anomaly in deg :

1. Mean anomaly in deg of the earth around sun, for input Julian day, year, month, day, hours, since Standard Epoch J2000

   Input UT year = 2000 month = 1 day = 1 hr = 12 min = 0 sec = 0.00000 Corresponds to julian_day = 2451545.0000000000

   Output Mean anomaly in deg of the earth around Sun = 357.5291000000, at input Julian day, minimizing into 0-360 deg = 357.5291000000

2. True anomaly in deg of the earth around sun, for input Julian day, year, month, day, hours, since Standard Epoch J2000

   Input UT year = 2000 month = 1 day = 1 hr = 12 min = 0 sec = 0.00000 Corresponds to julian_day = 2451545.0000000000

   Output True anomaly in deg of the earth around Sun = 357.4447876113, at input Julian day, minimizing into 0-360 deg = 357.4447876113

Note : Reported values are same, Mean anomaly of Earth around Sun g = 357.53, at Epoch J2000; **(Ref. Indian Astronomical Ephemeris, Year 2000, IMD, Page 528.)**

Next Section - 2.3 Earth orbit points - UT at perihelion and aphelion

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Earth Orbit Input Year : Precise Universal Time (UT) at orbit points - Perihelion and Aphelion.

Finding Universal Time (UT) at Perihelion and Aphelion point, Sensing parameter is Mean anomaly (ME) in deg.

Perihelion and Aphelion describes two specifc points on Earth orbit around Sun.

- perihelion is Point on orbit nearest to Sun, is about 147,098,074 km, and sensing parameter ME deg cross over 360 ie 0 deg around January 03.
- aphelion is Point on orbit farthest from Sun, is about 152,097,701 km, and sensing parameter ME_deg cross over 180 deg around July 04.

This difference in distance to sun, while earth is at perihelion or aphelion. However the difference not enough to affect the earth's climate.

Finding Precise time to reach Perihelion and Aphelion (using sensing parameter ME deg), for any Input year .

1. Find Precise time for Earth to reach Perihelion : Input Year = 2013

   Output Time at Perihelion : julian_day = 2456295.8832941712, ie year = 2013, month = 1, day = 3, hr = 9, min = 11, sec = 56.61639

2. Find Precise time for Earth to reach Aphelion : Input Year = 2013

   Output Time at Aphelion : julian_day = 2456478.5131087117, ie year = 2013, month = 7, day = 5, hr = 0, min = 18, sec = 52.59269

To verify computed Perihelion and Aphelion Time, apply them as input & compute back Mean anomaly, respectively expected values as 0 deg and 180 deg.

3. Find Mean anomaly and True anomaly in deg : Input Julian day = 2456295.8832941712 is precise time for Earth at Perihelion

   Output Mean anomaly in deg at Perihelion = 5040.0010049824 minimizing into 0-360 deg = 0.0010049824, note error in deg = 0.00100

   True anomaly in deg at Perihelion = 5040.0010392859 minimizing into 0-360 deg = 0.0010392859, note error in deg = 0.00104

4. Find Mean anomaly and True anomaly in deg : Input Julian day = 2456478.5131087117 is precise time for Earth at Aphelion

   Output Mean anomaly in deg at Aphelion = 5220.0010013299 minimizing into 0-360 deg = 180.0010013299, note error in deg = 0.00100

   True anomaly in deg at Aphelion = 5220.0009685492 minimizing into 0-360 deg = 180.0009685492, note error in deg = 0.00097

Note 1 : Perihelion & Aphelion time, reported, **(Ref. http://www.usno.navy.mil/USNO/astronomical-applications/data-services/earth-seasons).**

Perihelion, Year 2013, Month 1, Day 02, Hrs 05, UTC ; Aphelion, Year 2013, Month 7, Day 05, Hrs 15, UTC.

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Note 2 : For verifying, the anomaly values are calculated using analytical equations which are approximate, giving error at 3rd place of decimal;

The algorithmic solutions are offered in later sections that goes through many iterations minimizing the errors.

Next Section - 2.4 Earth orbit points - UT at vernal and autumnal equinox

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Earth Orbit Input Year : Precise Universal Time (UT) at orbit points - Vernal Equinox and Autumnal Equinox.

Finding Universal Time (UT) at Vernal equinox and Autumnal equinox point, Sensing parameter is Declination (delta).

Equinox occurs twice a year, when Earth rotation axis is exactly parallel to the direction of motion of Earth around Sun.

Vernal equinox around March 20/21, and Autumnal equinox around September 22/23; at equinox day and night are of same length.

Right Ascension (RA or alpha) and Declination (delta) are astronomical terms for coordinates of a point on celestial sphere.

Right Ascension (RA), is similar to longitude on Earth is measured in hours (h), minutes (m) and seconds (s).

RA around the celestial equator is 24 hours, where 1 h = 15 deg. Unlike longitude (zero deg) on Earth as Prime Meridian,

the reference Right Ascension (zero hour) is First Point of Aries in sky where Sun crosses celestial equator called Vernal equinox.

Declination (delta), is similar to latitude on Earth is measured in degrees, arc-minutes and arc-seconds.

Declination measures how far overhead an object will rise in the sky, measured 0 deg at the equator, +90 deg at North Pole and -90 deg at South Pole.

Vernal point : RA = 00h 00m 00s and longitude = 0 deg , the sense parameter Delta_deg sign change -ve to 0 to +ve

Autumnal point : RA = 12h 00m 00s and longitude = 180 deg, the sense parameter Delta_deg sign change +ve to 0 to -ve

Finding Precise time to reach Vernal_equinox and Autumnal_equinox (using sense parameter Delta_deg), for any Input year are as follows.

1. Find Precise time for Earth to reach Vernal equinox : Input Year = 2013

   Output Time at Vernal equinox : Julian day = 2456371.9598282082, ie year = 2013, month = 3, day = 20, hr = 11, min = 2, sec = 9.15719

2. Find Precise time for Earth to reach Autumnal equinox : Input Year = 2013

   Output Time at Autumnal equinox : Julian day = 2456558.3650290174, ie year = 2013, month = 9, day = 22, hr = 20, min = 45, sec = 38.50711

Note : Values reported are same, **(Ref. http://www.usno.navy.mil/USNO/astronomical-applications/data-services/earth-seasons).**

Vernal Equinox, Year 2013, Month 3, Day 20, Hrs 11, Min 02 UTC ; Autumnal Equinox, Year 2013, Month 9, Day 22, Hrs 20, Min 44 UTC.

Next Section - 2.5 Earth orbit points - UT at summer and winter solstice

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Earth Orbit Input Year : Precise Universal Time (UT) at orbit points - Summer Solstice and Winter Solstice .

Finding Universal Time (UT) at Summer and Winter Solstice point, Sensing parameter is Declination (delta).

Solstice occurs twice a year, as Sun appears to have reached its highest or lowest annual altitude in the sky at local solar noon.

Summer Solstice is around June 21/22, and Winter Solastice is around December 21/22.

Solstices, together with the Equinoxes, are connected with the seasons.

Summer Solstice : the sense parameter Delta_deg max about +23.44 deg

Winter Solstice : the sense parameter Delta_deg min about -23.44 deg

Finding Precise time to reach Summer and Winter Solstice (using sense parameter Delta_deg), for any Input year are as follows.

1. Find Precise time for Earth to reach Summer Solstice : Input Year = 2013

   Output Time at Summer Solstice : julian_day = 2456464.7092499998, ie year = 2013, month = 6, day = 21, hr = 5, min = 1, sec = 19.19999

2. Find Precise time for Earth to reach Winter Solstice : Input Year = 2013

   Output Time at Winter Solstice : julian_day = 2456648.2153210002, ie year = 2013, month = 12, day = 21, hr = 17, min = 10, sec = 3.73442

Note : Values reported is slightly less then 3 min and 1 min, (Ref. http://www.usno.navy.mil/USNO/astronomical-applications/data-services/earth-seasons).

Summer Solstice, Year 2013, Month 6, Day 21, Hrs 05, Min 04 UTC ; Winter Solstice, Year 2013, Month 12, Day 21, Hrs 17, Min 11 UTC.

Next Section - 2.6 Earth orbit points - UT at semi-major and semi-minor axis

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OM-MSS Section - 2.6 --19

Earth Orbit Input Year : Precise Universal Time (UT) at orbit points - Semi-Major Axis and Semi-Minor Axis .

Finding Universal Time (UT) at Semi-Minor Axis point and Semi-Major Axis point, Sensing parameter is Mean anomaly (ME) in deg .

Semi-Minor Axis Point around Apri 01, sensing parameter ME_deg cross over 90 deg,

Semi-Major Axis Point around July 05, sensing parameter ME_deg cross over 180 deg.

Finding Precise time to reach Semi-Minor Axis and Semi-Major Axis Point (using sensing parameter ME deg), for any Input year are as follows.

1. Find Precise time for Earth to reach Semi-Minor Axis : Input Year = 2013

   Output Time at Semi-Minor Axis Point : Julian day = 2456387.1982014412, ie year = 2013, month = 4, day = 4, hr = 16, min = 45, sec = 24.60452

2. Find Precise time for Earth to reach Semi-Major Axis : Input Year = 2013

   Output Time at Semi-Major Axis Point : Julian day = 2456478.5131087117, ie year = 2013, month = 7, day = 5, hr = 0, min = 18, sec = 52.59269

Next Section - 2.7 Earth orbit - astronomical years

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OM-MSS Section - 2.7 --20

Earth Orbit Input Year : Astronomical Years - Anomalistic, Tropical, and Sidereal Years .

The Anomalistic, Tropical, and Sidereal Years are Astronomical years. Look at the differeces.

Finding Anomalistic, Tropical, and Sidereal Years for the Input Year.

1. For Anomalistic year : find Precise time for Earth to reach Perihelion to Perihelion : Input Year = 2000

   Output Perihelion to Perihelion Time in julian_day = 365.2596290808 ie Days 365, hour 6, min 13, sec 51.95258 is Anomalistic year

2. For Anomalistic year : find Precise time for Earth to reach Aphelion to Aphelion : Input Year = 2000

   Output Aphelion to Aphelion Time in julian_day = 365.2596290810 ie Days 365, hour 6, min 13, sec 51.95260 is Anomalistic year

3. For Tropical year : find Precise time for Earth to reach Vernal to Vernal equinox : Input Year = 2000

   Output Vernal to Vernal equinox Time in julian_day = 365.2423121394 ie Days 365, hour 5, min 48, sec 55.76884 is Tropical year

4. For Tropical year : find Precise time for Earth to reach Autumnal to Autumnal equinox : Input Year = 2000

   Output Autumnal to Autumnal equinox Time in julian_day = 365.2423121394 ie Days 365, hour 5, min 48, sec 55.76884 is Tropical year

Note : Values reported are almost same, **(Ref. http://en.wikipedia.org/wiki/Year , http://www.yourdictionary.com/sidereal-year).**

Anomalistic year : Days 365, Hrs 06, Min 13, Sec 52.6 UTC ; Tropical year : Days 365, Hrs 05, Min 48, Sec 46 UTC .

Earth's one revolution around Sun called Sidereal year = 365.256363004 (Days 365, Hr 06, Min 09, Sec 09.76) in units of mean solar days, at epoch J2000

The difference in days among Anomalistic, Tropical and Sidereal Year are

(Anomalistic - Tropical) year = Days 0, hour 0, minute 24, seconds 56.1837373674

(Sidereal - Anomalistic) year = Days 0, hour 0, minute -4, seconds -42.1890329762

(Sidereal - Tropical) year = Days 0, hour 0, minute 20, seconds 13.9947043912

Next Section - 2.8 Earth orbit oblateness - semi-major and semi-minor axis

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Earth Orbit Oblateness : Semi-Major Axis and Semi-Minor Axis.

Finding Semi-Major Axis and Semi-Minor Axis in km.

The Earth's orbit is an ellipse. The Earth's shape is very close to an oblate spheroid, with a bulge around the equator.

GM_SUN is Gravitational parameter of Sun is product of gravitational constant G and mass M of Sun

GM_SUN = = 132,712,440,018 km3/sec2 = 132712.440018e6 = 132712.440018 x 10 to pow 6.

The Semi-Major Axis value is computed considering using earth mean motion rev per day obtain as

(a) EARTH mean motion rev per day = 0.0027377786, as 1.0 / 365.259629080 days is time Perihelion to Perihelion same as Aphelion to Aphelion, year 2000.

(b) EARTH mean motion rev per day = 0.0027378030, as 1.0 / 365.256363004 days is sidereal year for Earth making one full revolution around Sun.

The Semi-Minor Axis value is calculated considering earth orbit Eccentricity = 0.016710219

1. Earth Semi-Major Axis (SMA) in km, using EARTH mean motion rev per day as 0.0027377786 using diff. of Julian days Perihelion to Perihelion, year 2000

   (a) Ignoring Earth oblateness

   Input Earth mean motion rev per day = 0.0027377786, GM SUN = 132712440018.00000

   Output Semi Major Axis in km = 149598616.3114941400, and Semi Minor Axis in km = 149577728.5363029200

   (b) Considering Earth oblateness , Inclination , Eccentricity

   Input Earth mean motion rev per day = 0.0027377786, GM SUN = 132712440018.00, Incl = 23.43928, Ecc = 0.01671, constant_k2 = 65915.34460

   Output Semi-Major Axis in km = 149598616.3117182900, and Semi-Miror Axis in km = 149577728.5365270400

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2. Earth Semi-Major Axis (SMA) in km, using EARTH mean motion rev per day as 0.0027378030 obtained using sidereal year for Earth one revolution around Sun.

(a) Ignoring Earth oblateness

Input Earth mean motion rev per day = 0.0027378031, GM SUN = 132712440018.00

Output Semi-Major Axis in km = 149597724.5233797700, and Semi-Minor Axis in km = 149576836.8727048900

(b) Considering Earth oblateness , Inclination , Eccentricity

Input Earth mean motion rev per day = 0.0027378031, GM SUN = 132712440018.00, Incl = 23.43928, Ecc = 0.01671, constant_k2 = 65915.34460

Output Semi-Major Axis in km = 149597724.5236039200, and Semi-Miror Axis in km = 149576836.8729290100

Note : Compare with Two different values of Semi-Major Axis reported as

Semi-major axis = 149,597,887.5 KM , **(Ref. http://simple.wikipedia.org/wiki/Earth)**

Semi-major axis = 149,598,261 KM , **(Ref. http://en.wikipedia.org/wiki/Earth 27s_orbit)**

Next Section - 2.9 Earth orbit - mean, eccentric and true anomaly at UT

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Earth Orbit Input Year : Mean anomaly, Eccentric anomaly, True anomaly at UT, based on algorithms of iterative method.

Finding Mean anomaly, Eccentric anomaly, True anomaly at any UT, year, month, day, hour, minute, seconds.

The Mean anomaly and True anomaly values presented before, were calculated using standard analytical expressions in section 2.2.

Here the anomalies are computed based on algorithms of iterative method, while Earth moves through the respective orbit points :

perihelion, vernal equinox, semi_minoraxis, summer solstice, aphelion, autumnal equinox, winter solstice.

The Computed values of Mean, Eccentric, and True anomaly mentioned below show accuracy of the algorithms.

The Input Time at each orbit point is same as what were computed before in sections (2.3 to 2.6).

Mean anomaly, Eccentric anomaly, True anomaly at Perihelion, Aphelion, Equinoxes, Solstices & Semi-minor axis points :

1. Input time at Perihelion : year = 2013, month = 1, day = 3, hour = 9, minute = 11, seconds = 56.6163906455

   Output the Anomalies in deg : Mean anomaly = 0.00000, Eccentric anomaly = 0.00000, True anomaly = 0.00000

2. Input time at Vernal equinox : year = 2013, month = 3, day = 20, hour = 11, minute = 2, seconds = 9.1571885347

   Output the Anomalies in deg : Mean anomaly = 74.98105, Eccentric anomaly = 75.90967, True anomaly = 76.84023

3. Input time at Summer solstice : year = 2013, month = 6, day = 21, hour = 5, minute = 1, seconds = 19.1999861598

   Output the Anomalies in deg : Mean anomaly = 166.39491, Eccentric anomaly = 166.61653, True anomaly = 166.83637

4. Input time at Aphelion : year = 2013, month = 7, day = 5, hour = 0, minute = 18, seconds = 52.5926899910

   Output the Anomalies in deg : Mean anomaly = 180.00000, Eccentric anomaly = 180.00000, True anomaly = 180.00000

5. Input time at Autumnal equinox : year = 2013, month = 9, day = 22, hour = 20, minute = 45, seconds = 38.5071069002

   Output the Anomalies in deg : Mean anomaly = 258.70208, Eccentric anomaly = 257.76639, True anomaly = 256.83232

6. Input time at Winter solstice : year = 2013, month = 12, day = 21, hour = 17, minute = 10, seconds = 3.7344172597

   Output the Anomalies in deg : Mean anomaly = 347.25855, Eccentric anomaly = 347.04389, True anomaly = 346.82746

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7. Input time at Semi-minor axis : year = 2013, month = 4, day = 4, hour = 16, minute = 45, seconds = 24.6045202017

   Output the Anomalies in deg : Mean anomaly = 90.00000, Eccentric anomaly = 90.95729, True anomaly = 91.91449

8. The Semi-Major Axis Point is same as Aphelion presented above, repeated here for completeess.

   Input time at Semi-major axis : year = 2013, month = 7, day = 5, hour = 0, minute = 18, seconds = 52.5926899910

   Output the Anomalies in deg : Mean anomaly = 180.00000, Eccentric anomaly = 180.00000, True anomaly = 180.00000

Note : The values computed above are based on iterative algorithm, therefore more accurate and validated against the expected respective values of anomaly as 0 deg, 180 deg, and 90 deg at Perihelion, Aphelion & Semi-minor axis points.

The values at Equinoxes and Solstices are also close to those reported mentioned before.

Next Section - 2.10 Earth orbit - four seasons

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Earth Orbit Input Year : Four Seasons - Spring, Summer, Autumn, and Winter.

Finding Four Seasons : Start Time of Spring, Summer, Autumn, Winter.

Seasons are a subdivision of a year. The Earth's rotation axis is tilted by 23.4392794383 degrees with respect to the ecliptic.

Because of this tilt in Earth's rotation axis, the Sun appears at different elevations or angle above the horizon, at different times of a year.

The variation in the elevation of the Sun over the year is the cause of the seasons.

The 0 to 360 deg Sun's longitudes are equally divided among four seasons as :

Sun longitudes : 0 - 90 deg (Spring), 90 - 180 deg (Summer), 180 - 270 deg (Autumn), 270 - 360 deg (Winter).

The Sun true longitude (Lsun) is derived from Sun mean longitude(Lmean) and Earth mean anomaly(ME).

For start time of any season, the sensing parameter is Sun true longitude value which is reached through many iterations.

Finding the Start Time of the Seasons - Spring, Summer, Autumn, Winter, and the corresponding Sun true Longitude (Lsun) for the input year

1. Year = 2013, Start Time of Spring : UT year = 2013, month = 3, day = 20, hr = 11, min = 2, sec = 9.15719, & Sun true Log deg = 360.00000

2. Year = 2013, Start Time of Summer : UT year = 2013, month = 6, day = 21, hr = 5, min = 1, sec = 23.88007, & Sun true Log deg = 90.00000

3. Year = 2013, Start Time of Autumn : UT year = 2013, month = 9, day = 22, hr = 20, min = 45, sec = 38.50711, & Sun true Log deg = 180.00000

4. Year = 2013, Start Time of Winter : UT year = 2013, month = 12, day = 21, hr = 17, min = 10, sec = 7.88032, & Sun true Log deg = 270.00000

Finding the Duration of the Seasons - Spring, Summer, Autumn, Winter for the Year = 2013

1. Season Spring Duration Days 92, hour 17, minute 59, seconds 14.72288
2. Season Summer Duration Days 93, hour 15, minute 44, seconds 14.62704
3. Season Autumn Duration Days 89, hour 20, minute 24, seconds 29.37321
4. Season Winter Duration Days 88, hour 23, minute 40, seconds 57.24261

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Summary of four seasons, Year 2013 , first day of Spring, Summer, Autumn, and Winter season.

- 1st day of Spring season, Mar. 20, Vernal equinox, Sun crosses Equator moving northward, is beginning of a long period of sunlight at Pole.
- 1st day of Summer season, Jun. 21, Summer solstice, Sun is farthest north and time between Sunrise and Sunset is longest of the year,
- 1st day of Autumn season, Sept. 22, Autumnal equinox, Sun crosses Equator moving southward, is beginning of a long period of darkness at Pole.
- 1st day of Winter season, Dec. 21, Winter solstice, Sun is farthest south and time between Sunrise and Sunset is shortest of the year.

Note : Values reported are same, **(Ref. http://en.wikipedia.org/wiki/Season).**

Spring season (Vernal equinox) Year 2013, Mar. 20, Hrs 11, Min 02 UTC ; Summer season (Summer solstice) Year 2013, Jun. 21, Hrs 05, Min 04 UTC.

Autumn season (Autumnal equinox) Year 2013, Sept. 22, Hrs 20, Min 44 UTC ; Winter season (Winter solstice) Year 2013, Dec. 21, Hrs 17, Min 11 UTC.

Next Section - 2.11 Concluding astronomical events

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OM-MSS Section - 2.11 --24

Concluding Astronomical Events Anomalies, Equinoxes, Solstices, Years & Seasons presented in Sections (2.0 to 2.10).

In Sections (2.1 to 2.10), all that presented were prediction / computation of following Astronomical Time Events of Earth Orbit around Sun :

1. Precise value for Mean, Eccentric & True anomalies;
2. Precise time for Earth to reach Perihelion & Aphelion points;
3. Precise time for Earth to reach Vernal & Autumnal equinox points;
4. Precise time for Earth to reach Summer & Winter solstice points;
5. Precise time for Earth to reach Semi-major & Semi-major axis points;
6. Duration of Anomalistic, Tropical & Sidereal years;
7. Start time & durations of seasons - Spring, Summer, Autumn, & Winter.

End of Computing Astronomical Events Anomalies, Equinoxes, Solstices, Years & Seasons.

Next Section - 3 Position of Sun on Celestial Sphere at UT

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ANNEXURE : A Collection of few OM-MSS related Diagrams / Help.

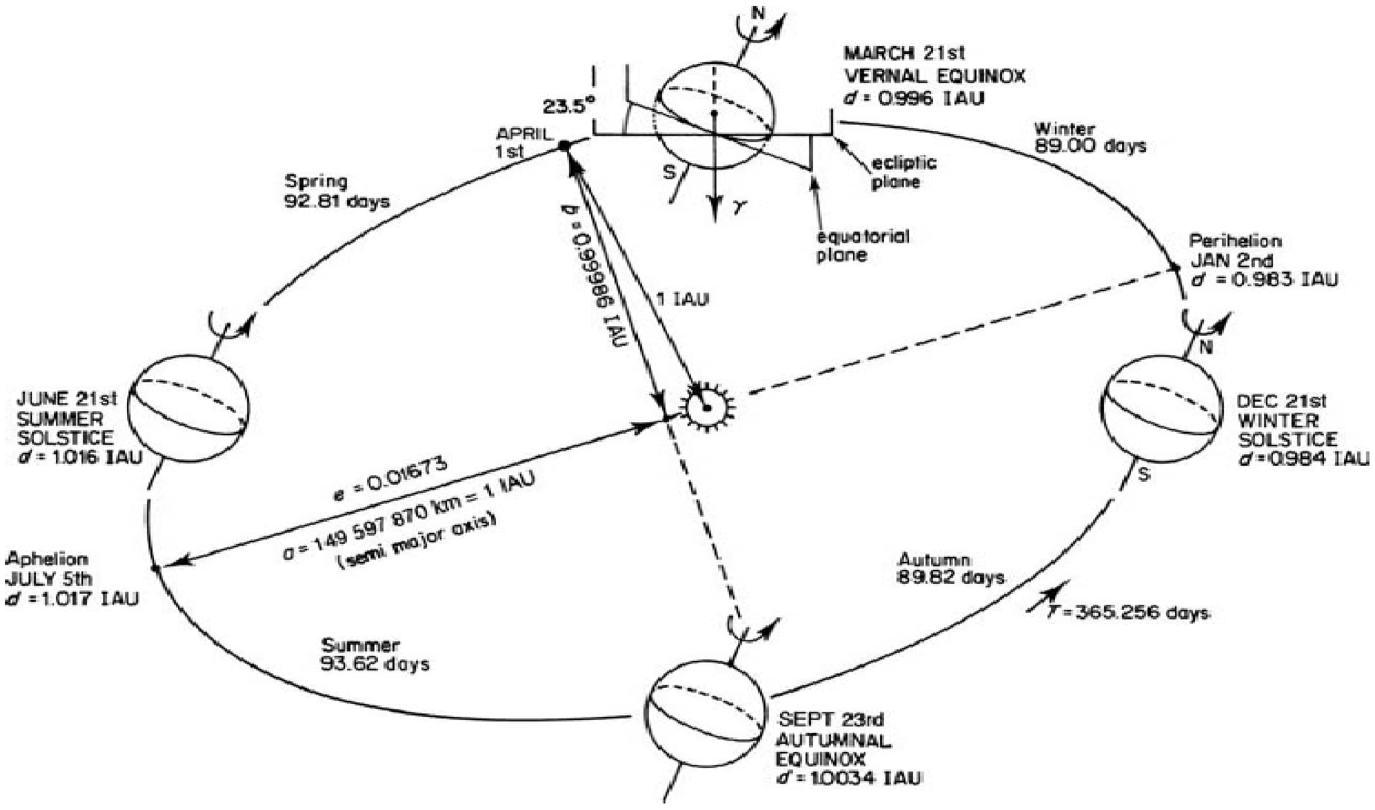

Fig-3. Orbit of Earth Around Sun

Earth rotates around sun with a period of approximately 365.25 days following an Ellipse of Eccentricity 0.01673 and Semi-major axis 149597870 km, which defines the Astronomical unit of distance (AU). Around 2 January, Earth is nearest from sun called Perihelion while around 5 July it is farthest from Sun called Aphelion (around 152100000 km). The other events point are Vernal equinox around 21 March, Autumnal equinox around 23 September, Summer solstice around 21 June and Winter solstic around 21 December. The plane of the orbit is called the plane of the Ecliptic that makes an angle 23.44 deg (the Obliquity of the Ecliptic) with the mean Equatorial plane.

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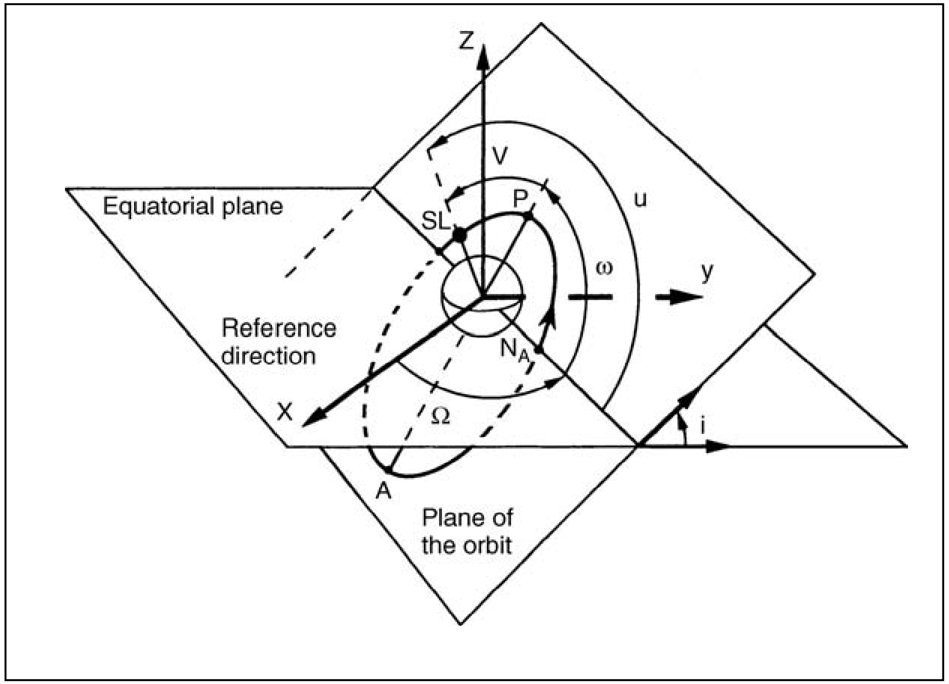

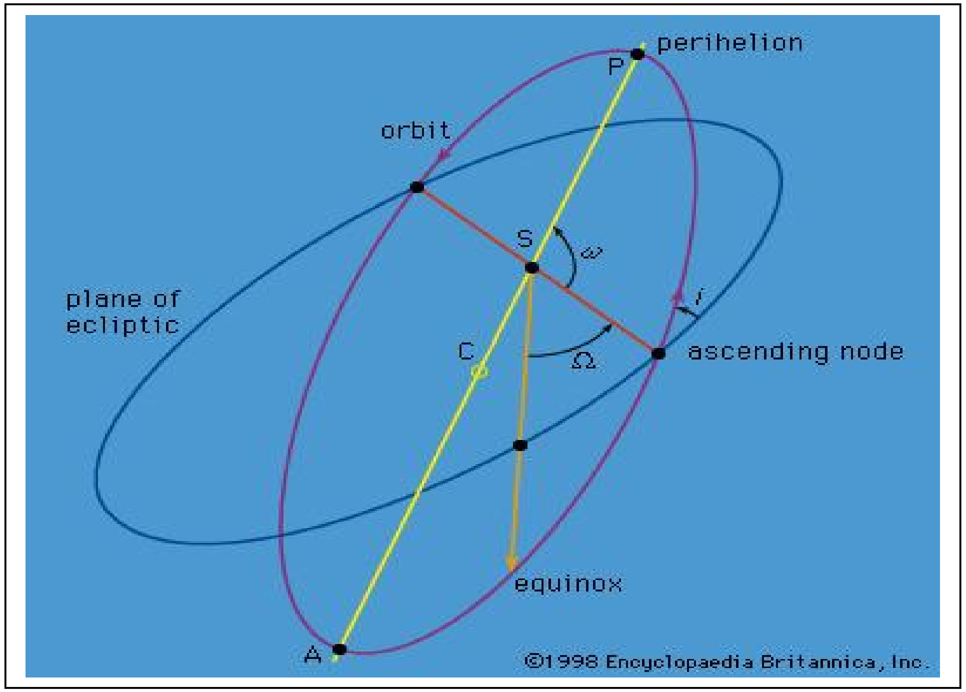

Fig- 4 & 5 Positioning of Orbit in Space

Orbit Position in Space at Epoch is defined by the Values of Kepler Orbit elements : (definations apply to both planets & Satellits)

1. Inclination 'i' of the orbit of a planet, is angle between the plane of planet's orbit and the plane containing Earth's orbital path (ecliptic).
2. Right ascension 'Ω' of the ascending node is the angle taken positively from 0 to 360 deg in the forward direction, between the reference direction and the ascending node of the orbit (the intersection of the orbit with the plane of the equator crossing this plane from south to north).
3. Argument of Perigee 'ω', specify angle between orbit's perigee and orbit's ascending node, measured in orbital plane and direction of motion.
4. Eccentricity '*e*' of an orbit shows how much the shape of an object's orbit is different from a circle;
5. Mean Anomaly '*v*' relates the position and time for a body moving in a Kepler orbit. The mean anomaly of an orbiting body is the angle through which the body would have traveled about the center of the orbit's auxiliary circle. 'M' grows linearly with time.

A knowledge of above five parameters completely defines the trajectory of an object or satellite in space. However, the Nodal angular elongation '*u*' can also be used to define the position of the satellite in its orbit. This is the angle taken positively in the direction of motion from 0 to 360 deg between the direction of the ascending node and the direction of the satellite ***(u = ω + v).***

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